Nr. 24.) $p(x) = x^5 - 2x^3 + 9x^2 - 3x + 4$

$x = (x+2) - 2 \qquad x^2 = [(x+2) - 2]^2 = (x+2)^2 - 4(x+2) + 4$

$x^3 = [(x+2)-2]^3 = (x+2)^3 - 6(x+2)^2 + 6(x+2) - 8$

$x^5 = [(x+2)-2]^5 = (x+2)^5 - 10(x+2)^4 + 40(x+2)^3 - 80(x+2)^2$
$+ 80(x+2) - 32$

$\Rightarrow p(x) = (x+2)^5 - 10(x+2)^4 + 38(x+2)^3 - 59(x+2)^2 + 17(x+2) + 30$

$q(x) = x^4 + x - 3 \qquad q(3) = ?$

1	0	0	1	-3
0	3	9	27	84
1	3	9	28	81

$\Rightarrow q(3) = 81$

$(x^5 - 2x^3 + 9x^2 - 3x + 4) : (x^4 + x - 3) = x + \frac{-2x^3 + 8x^2 + 4}{x^4 + x - 3}$

$-x^5 \qquad -x^2 + 3x$

$-2x^3 + 8x^2 \qquad +4$

Nr 25.)

$$p(x) = 2 \cdot \frac{(x+2)(x+1)x(x-1)(x-2)(x-3)}{(-3+2)(-3+1)(-3)(-3-1)(-3-2)(-3-3)} +$$

$$1 \cdot \frac{(x+3)(x+1)x(x-1)(x-2)(x-3)}{(-2+3)(-2+1)(-2)(-2-1)(-2-2)(-2-3)} +$$

$$(-1) \cdot \frac{(x+3)(x+2)x(x-1)(x-2)(x-3)}{(-1+3)(-1+2)(-1)(-1-1)(-1-2)(-1-3)} +$$

$$(-1) \frac{(x+3)(x+2)(x+1)(x-1)(x-2)(x-3)}{(0+3)(0+2)(0+1)(0-1)(0-2)(0-3)} +$$

$$1 \cdot \frac{(x+3)(x+2)(x+1)x(x-2)(x-3)}{(1+3)(1+2)(1+1) \, 1 \, (1-2)(1-3)} +$$

$$3 \cdot \frac{(x+3)(x+2)(x+1)x(x-1)(x-3)}{(2+3)(2+2)(2+1) \, 2 \, (2-1)(2-3)} +$$

$$4 \cdot \frac{(x+3)(x+2)(x+1)x(x-1)(x-2)}{(3+3)(3+2)(3+1) \, 3 \, (3-1)(3-2)} =$$

$$\ldots = -1 + \frac{37}{30}x + \frac{197}{180}x^2 - \frac{1}{4}x^3 - \frac{7}{72}x^4 - \frac{1}{60}x^5 + \frac{1}{360}x^6$$

Nr 27.)

a.) $\sinh(x+y) = \frac{1}{2}\left(e^{x+y} - e^{-x-y}\right)$

$\cosh x \sinh y = \frac{1}{4}\left(e^{x} + e^{-x}\right)\left(e^{y} - e^{-y}\right) =$

$= \frac{1}{4}\left(e^{x+y} - e^{x-y} + e^{y-x} - e^{-x-y}\right)$

$\sinh x \cosh y = \frac{1}{4}\left(e^{x} - e^{-x}\right)\left(e^{y} + e^{-y}\right) =$

$\frac{1}{4}\left(e^{x+y} + e^{x-y} - e^{-x+y} - e^{-x-y}\right)$

$\cosh x \sinh y + \sinh x \cosh y = \frac{1}{2}\left(e^{x+y} - e^{-x-y}\right)$ ✓

Nr 26 b.) $\frac{1}{2}\left(\ln(x^2-1)\right) - \ln(x+1) = 1$

$\ln\left(\sqrt{x^2-1}\right) - \ln(x+1) = 1$

$\ln\left(\frac{\sqrt{x^2-1}}{x+1}\right) = 1$

$\frac{\sqrt{(x+1)(x-1)}}{x+1} = e$

$\frac{\sqrt{(x+1)(x-1)}}{\sqrt{(x+1)^2}} = e$

$\sqrt{\frac{x-1}{x+1}} = e$

$\frac{x-1}{x+1} = e^2 \Rightarrow x(e^2-1) = -1-e^2$

$\Rightarrow x = \frac{1+e^2}{1-e^2}$

Nr 26 b)

Es gilt $x = \frac{1+e^2}{1-e^2} < -1$

Da die Arcosh nur für $x \geq 1$ definiert

$\Rightarrow L = \emptyset$

Nr 27 a.)

$\cosh(2x) = 2\cosh^2 x - 1$

$\frac{1}{2}\left(e^{2x} + e^{-2x}\right) = 2 \cdot \left[\frac{1}{4} \cdot \left(e^{x} + e^{-x}\right)^2\right] - 1$

$= \frac{1}{2}\left[e^{2x} + 2 + e^{-2x}\right] - 1$

$= \frac{1}{2}\left(e^{2x} + e^{-2x}\right)$ ✓

Nr 26 a.) $e^{2x} + 3e^{x} - \ln e^{3} \geq 1$

$e^{2x} + 3e^{x} - 4 \geq 0$ $\qquad e^{x} = u$

$u^2 + 3u - 4 \geq 0$

$u_{1,2} = \frac{-3 \pm \sqrt{9+16}}{2} = \frac{-3 \pm 5}{2} \Rightarrow u_1 = -4$

$u_2 = 1$

$\Rightarrow L = \{ u \in \mathbb{R} \mid u \geq 1 \ \vee \ u \leq -4 \}$

$\Rightarrow L = \{ x \in \mathbb{R} \mid e^{x} \geq 1 \ \vee \ e^{x} \leq -4 \} =$

$= \{ x \in \mathbb{R} \mid x \geq 0 \}$

Nr. 26 c.)

$$\ln(e^x+1)+\ln\left(e^x-\frac{1}{2}\right)=x$$

$$\ln\left[(e^x+1)\left(e^x-\frac{1}{2}\right)\right]=x$$

$$\ln\left(e^{2x}+\frac{1}{2}e^x-\frac{1}{2}\right)=x$$

$$e^{2x}+\frac{1}{2}e^x-\frac{1}{2}=e^x$$

$$e^{2x}-\frac{1}{2}e^x-\frac{1}{2}=0 \qquad e^x=u$$

$$u^2-\frac{1}{2}u-\frac{1}{2}=0$$

$$u_{1,2}=\frac{\frac{1}{2}\pm\sqrt{\frac{1}{4}+2}}{2}=$$

$$u_{1/2}=\frac{\frac{1}{2}\pm\frac{3}{2}}{2} \Rightarrow u_1=1$$

$$\left(u_2=-1/2\right)$$

$$\Rightarrow e^x=1$$

$$L=\{x\in\mathbb{R} \mid x=0\}$$

Nr 28 a.)

$$\sin x + \cos(2x) - 3\cos^2 x = 0$$

$$\sin x + (\cos^2 x - \sin^2 x) - 3\cos^2 x = 0$$

$$\sin x - 2\cos^2 x - \sin^2 x = 0$$

$$\sin x - 2[1 - \sin^2 x] - \sin^2 x = 0$$

$$\sin x - 2 + \sin^2 x = 0 \qquad \sin x = u$$

$$u^2 + u - 2 = 0$$

$$u_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2} \Rightarrow u_1 = \frac{-1+3}{2} = 1$$

$$\left(u_2 = \frac{-1-3}{2} = -2\right)$$

$$L = \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{2} + 2k\pi \quad k \in \mathbb{Z} \right\}$$

b.) $\sin(2x) + 3\sin x - 2\tan x = 0$

$$2\sin x\cos x + 3\sin x - 2\frac{\sin x}{\cos x} = 0$$

$$\sin x\left(2\cos x + 3 - \frac{2}{\cos x}\right) = 0$$

1. Fall: $\sin x = 0 \Rightarrow L_1 = \{ x \in \mathbb{R} \mid x = k\pi,\ k \in \mathbb{Z} \}$

2. Fall: $2\cos^2 x + 3\cos x - 2 = 0 \qquad \cos x = u$

$$2u^2 + 3u - 2 = 0 \qquad u_{1,2} = \frac{-3 \pm \sqrt{9+16}}{4}$$

$$\Rightarrow u_1 = \frac{-3+5}{4} = \frac{1}{2}$$

$$(u_2 = -2)$$

$$\Rightarrow L_2 = \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{3} + k2\pi \ \vee \ x = -\frac{\pi}{3} + k2\pi \right\}$$

$$L = L_1 \cup L_2$$