

$$A = \begin{pmatrix} 3 & 0 & 4 \\ 0 & 5 & 0 \\ 1 & 0 & 3 \end{pmatrix}$$

$$\det |A - \lambda E| = 0$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$\begin{vmatrix} 3-\lambda & 0 & 4 \\ 0 & 5-\lambda & 0 \\ 1 & 0 & 3-\lambda \end{vmatrix} = (3-\lambda)(5-\lambda)(3-\lambda) + 4(-1(5-\lambda)) = \\ = (5-\lambda) \underbrace{((3-\lambda)^2 - 4)}_{=} = (5-\lambda)(3-\lambda-2)(3-\lambda+2) \\ = (5-\lambda)(-\lambda+1)(5-\lambda) = 0 \quad \lambda_1 = 5 \quad \lambda_2 = 1 \quad \lambda_3 = 5$$

$$\text{alg. Viel.}(1) = 1 \quad \text{alg. V.}(5) = 2$$

$\lambda_2 = 1$ Eigenraum

$$(A - 1 \cdot E) \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & 4 \\ 0 & 4 & 0 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$\begin{array}{l} \text{IV} \\ \text{II} \end{array} \quad 4y = 0 \quad \Rightarrow \quad y = 0$$

$$x + 2z = 0 \quad \Rightarrow \quad x = -2z$$

$$\begin{pmatrix} -2z \\ 0 \\ z \end{pmatrix} \quad z \in \mathbb{R}$$

$$\left\{ \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \cdot z : z \in \mathbb{R} \right\}$$

geom. V. $(\lambda_2) = 1$

$$\lambda_1 = \lambda_3 = 5$$

$$(A - 5E) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{III} \cdot \quad x - 2z = 0$$

$$x = 2z$$

y beliebig

$$\left\{ \begin{pmatrix} 2z \\ y \\ z \end{pmatrix} : y, z \in \mathbb{R} \right\} =$$

$$\left\{ \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} z + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot y : z, y \in \mathbb{R} \right\}$$

geom. Viel(λ_1) = 2

