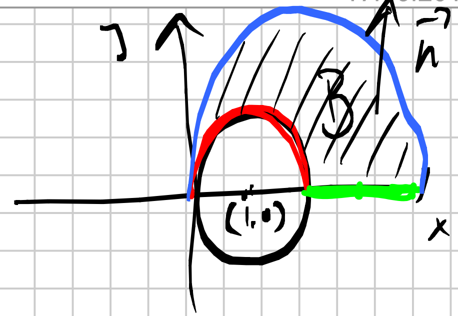
Tut Math II SS 2011 Bsp 1

Notiztitel 17.06.2011

$$\oint_{\varphi} \langle \vec{F}, d\vec{s} \rangle$$

$$\vec{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



(i) $y = \sqrt{2x - x^2} \qquad 0 \leq x \leq 2$

$$y^2 = 2x - x^2 \Rightarrow y^2 + x^2 - 2x = 0 \Rightarrow y^2 + \underbrace{x^2 - 2x + 1}_{(x-1)^2} = 1$$

$$y^2 + (x-1)^2 = 1$$

$$y \geq 0$$

(ii) $y = 0 \qquad 2 \leq x \leq 4$

(iii) $y = \sqrt{4x - x^2} \qquad 0 \leq x \leq 4$

$$y^2 + \underbrace{x^2 - 4x + 4}_{(x-2)^2} = 4$$

(iii) $y^2 + (x-2)^2 = 2^2$

$$\oint \langle \vec{F}, d\vec{s} \rangle = \iint_O \langle \operatorname{rot} \vec{F}, \vec{n} \rangle \, dO \qquad \text{STOKES}$$

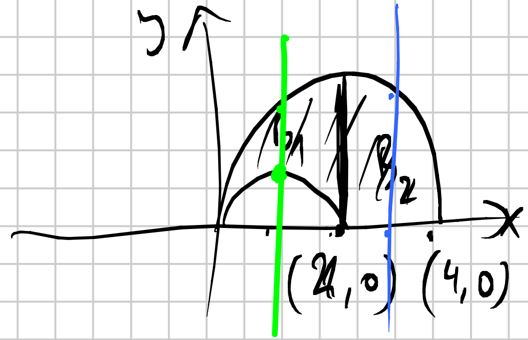
$\underline{\gamma = \partial(O)}$

Berechnen

$\operatorname{rot} \vec{F} \overset{?}{=}$

$$\left\| \begin{pmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy^2 & xy & 1 \end{pmatrix} \right\| = \vec{e}_1 \overbrace{\left(\frac{\partial}{\partial y} 1 - \frac{\partial}{\partial z}(xy) \right)}^{0} + \vec{e}_2 \underbrace{\left(\partial/\partial z \, xy^2 - \partial/\partial x \, 1 \right)} + $$

$$+ \vec{e}_3 \left(\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial y}\overset{0}{(xy^2)} \right)$$



$$\operatorname{rot}\vec{F} = \begin{pmatrix} 0 \\ 0 \\ y - 2xy \end{pmatrix}$$

$$\iint_{\Phi} \left\langle \begin{pmatrix} 0 \\ 0 \\ y-2xy \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle do = \iint_{B} (y-2xy)\, do =$$

$$= \iint_{B_1} (y-2xy)\, do + \iint_{B_2} (y-2xy)\, do = \int_0^2 \int_{\sqrt{2x-x^2}}^{\sqrt{4x-x^2}} y(1-2x)\, dy\, dx$$

$$+ \int_2^4 \int_0^{\sqrt{4x-x^2}} y(1-2x)\, dx\, dy = \int_0^2 (1-2x) \frac{y^2}{2} \Bigg|_{\sqrt{2x-x^2}}^{\sqrt{4x-x^2}} dx + \int_2^4 (1-2x) \frac{y^2}{2} \Bigg|_0^{\sqrt{4x-x^2}} dx$$

$$= \int_0^2 \left(\frac{(1-2x)}{2} \left(4x - \cancel{x^2} - 2x + \cancel{x^2}\right) \right) dx + \int_2^4 \frac{1-2x}{2} \left(4x - x^2\right) dx$$

$$= \int_0^2 (1-2x)\,x \; dx + \int_2^4 \frac{(1-2x)\,(4x-x^2)}{2}\, dx =$$

$$= \left(\frac{x^2}{2} - \frac{2x^3}{3} \right) \Bigg|_0^2 + \frac{1}{2} \int_2^4 \left(2x^3 - 9x^2 + 4x\right) dx =$$

$$= \quad - \quad + \frac{1}{2} \left(\frac{2x^4}{4} - \frac{9x^3}{3} + \frac{4x^2}{2} \right) \Bigg|_2^4 = -\frac{46}{3}$$