Lösung des 4. Bsp vom Tutorium am 27.5.2011

Notiztitel 26.05.2011

$$\left(2xy^2 + ye^x\right)dx + \left(x^2y - 1\right)dy = 0$$

$$A(xy)\,dx + B(xy)\,dy = 0$$

Integrabilitätsbedingung: $A_y = B_x$

$$A_y = 4xy + e^x \neq B_x = 2xy$$ also nicht erfüllt

daher DGL nicht exakt

Suche Integrierenden Faktor μ

$$\frac{B_x - A_y}{A} = -\frac{2xy + e^x}{2xy^2 + ye^x} = -\frac{1}{y} \Rightarrow \mu(y) = \exp\left(\int \frac{1}{y}dy\right)$$

$$\mu(y) = \exp\left(\int -\frac{1}{y}\,dy\right) = \exp\{-\ln|y| + c\} = \tilde{c}\,\frac{1}{|y|} = \frac{c}{y} \quad y \neq 0$$

Äquivalente exakte DGL:

$$\underbrace{\frac{2xy^2 + ye^x}{y}}_{A'}\,dx + \underbrace{\frac{x^2y - 1}{y}}_{B'}\,dy = 0 \quad \text{mit } A'_y = B'_x$$

Lösung: $\quad F_x = A' = 2xy + e^x$

$$F = \int A'(x,y)\,dx + \varphi(y) = \int (2xy + e^x)\,dx + \varphi(y)$$

$$F = x^2y + e^x + \varphi(y) \quad F_y = x^2 + \varphi'(y) = B' = \frac{x^2y - 1}{y}$$

$$F_y = x^2 + \varphi'(y) = \frac{x^2y - 1}{y} \Rightarrow \varphi'(y) = \frac{x^2y-1}{y} - \frac{x^2y}{y}$$

$\Rightarrow \varphi'(y) = -1/y$ (für $y > 0$; für $y < 0$ analog $\varphi(y) = \frac{1}{y}$)

$$\varphi(y) = \int -\frac{dy}{y} + c = \ln|y| + c = \ln y + c \quad (y > 0)$$

Allg. Lösung $F(x,y) = c$ $\quad x^2y + e^x + \ln y = c$

(Analog für $y < 0$ $\quad F(x,y) = c = x^2y + e^x + \ln(-y)$)

Also gesamt: $\quad x^2y + e^x + \ln|y| = c$

AWP: $y(1) = 1 \Rightarrow 1 + e^1 + \ln 1 = c \Rightarrow c = 1 + e$

$$\boxed{x^2y + e^x + \ln y = 1 + e}$$