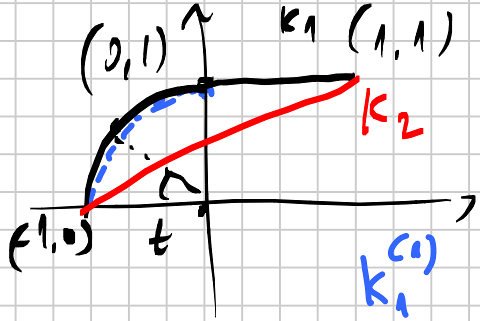
Tut. 27.5.2011 Bsp. 1

Notiztitel 27.05.2011

k_1, k_2 Kurven in $\mathbb{R}^2$

(a) $\vec{v}_1 = \begin{pmatrix} x \\ y \end{pmatrix}$ (b) $\vec{v}_2 = \begin{pmatrix} e^{\pi x} \cos(\pi y) \\ -e^{\pi x} \sin(\pi y) \end{pmatrix}$



(a) $\int_{k_1} (x\,dx + y\,dy) = \int_{k_1^{(1)}} (\quad) + \int_{k_1^{(2)}} (\quad)$

$k_1^{(1)}$ $x(t) = -\cos t$ $\quad \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}$

$y(t) = \sin t$

Parameter Winkel $t \in [0, \frac{\pi}{2}]$

$k_1^{(2)}$ Parameter x Koord.

$0 \le t \le 1$

$x(t) = t \quad \begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$y(t) = 1$

$$\int_{\substack{t=0\\k_1^{(1)}}}^{\frac{\pi}{2}} \Big(\underbrace{-\cos t}_{x} \underbrace{\sin t}_{\dot{x}}\, dt + \underbrace{\sin t}_{y} \underbrace{\cos t}_{\dot{y}}\, dt \Big) = 0 \qquad \int_{k_1^{(2)}} \ldots = \int_0^1 \big(t\, dt + 1 \cdot 0 \cdot dt \big) = \frac{t^2}{2}\Big|_0^1 = \frac{1}{2}$$

$$\int_{k_1} \ldots = 0 + \frac{1}{2} = \frac{1}{2}$$

Parametrierung k_2

$$x(t) = -1 + 2t \qquad \dot{x} = 2$$

$$y(t) = t \qquad \dot{y} = 1$$

$$\int_{k_2} y\,dx + y\,dy = \int_0^1 \big((-1+2t) \cdot 2\, dt + t \cdot 1 \cdot dt \big) = \ldots = \frac{1}{2}$$

$$\int_{k_1} \ldots = \int_{k_2} \ldots$$

Ist v_1 ein Gradientenfeld? Ja

$$\frac{\partial v_1^{(1)}}{\partial y} = \frac{\partial v_1^{(2)}}{\partial x} = 0$$

Ausrechnen der Potentials φ:

$$\varphi_x = \vec{v}_1^{(1)} = x \qquad \varphi_y = \vec{v}_1^{(2)} = y$$

$$\varphi = \int x\,dx + \psi(y) = \frac{x^2}{2} + \psi(y)$$

$$\varphi'_y = \psi'(y) = y \Rightarrow \psi(y) = \int y\,dy = \frac{y^2}{2} + k$$

$$\varphi(xy) = \frac{x^2}{2} + \frac{y^2}{2} + \underline{\text{konst}} \qquad \underline{\text{konst} = 0} \text{ Auswählen}$$

$$\varphi(xy) = \frac{x^2+y^2}{2} \qquad \int_{k_1} \dots = \int_{k_2} \dots = \varphi(1,1) - \varphi(-1,0) = 1/2$$

(b) $\vec{v}_2 = \begin{pmatrix} e^{\bar{u}x}\cos(\bar{u}y) \\ -e^{\bar{u}x}\sin(\bar{u}y) \end{pmatrix}$ $\exists A$

Ist $\vec{v}_2$ Gradientenfeld?

$$\frac{\partial\left(e^{\bar{u}x}\cos\bar{u}y\right)}{\partial y} = \frac{\partial\left(-e^{\bar{u}x}\sin(\bar{u}y)\right)}{\partial x}$$

$$-\bar{u}e^{\bar{u}x}\sin(\bar{u}y) = -\bar{u}e^{\bar{u}x}\sin(\bar{u}y) \checkmark$$

Berechne Potential: $\varphi(xy)$ sodass $\varphi'_x = e^{\bar{u}x}\cos(\bar{u}y)$

u. $\varphi'_y = -e^{\bar{u}x}\sin(\bar{u}y)$

$$\varphi = \int e^{\pi x} \cos(\pi y)\, dx + \psi(y) = \frac{1}{\pi} \cos(\pi y)\, e^{\pi x} + \psi(y)$$

$$\varphi'_y = -\sin\cancel{(\pi y)\, e^{\pi x}} + \psi'(y) \overset{!!}{=} -e^{\pi x} \cancel{\sin}(\pi y)$$

$$\psi'(y) = 0 \Rightarrow \psi(y) = \text{konst.}$$

$$\varphi = \frac{1}{\pi} \cos(\pi y)\, e^{\pi x} + \text{konst} \qquad \text{Wähle konst} = 0$$

$$\varphi = \frac{1}{\pi} \cos(\pi y)\, e^{\pi x} \qquad \int_{k_1} \ldots = \int_{k_2} \ldots = \varphi(1,1) - \varphi(-1,0) = -\frac{e^{\pi} + e^{-\pi}}{\pi} \;//$$