Bsp 1 20.5.2011

C: $r = \cos^2\varphi$ $\quad \varphi \in [0, 2\pi]$

a) Skizze

$\cos(\varphi) = \cos(\pi - \varphi)$

$\cos(\varphi) = -\cos(\pi + \varphi)$

$\cos(\varphi) = \cos(2\pi - \varphi)$

$\Rightarrow$ Symmetrisch bzgl x-Achse, y-Achse

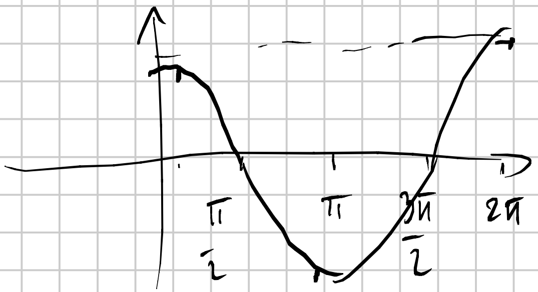
$(r,\varphi) \in C$

$\Downarrow$

$(r, \pi - \varphi) \in C$

$(r, \pi + \varphi) \in C$

$(r, 2\pi - \varphi) \in C$



$\underline{x = r\cos\varphi = \cos^3\varphi} \quad \cos\varphi = \sqrt[3]{x}$

$y = r\sin\varphi = \cos^2\varphi \sin\varphi = \cos^2\varphi\sqrt{1-\cos^2\varphi}$

$\cos^2\varphi = x^{2/3}$

$$y = x^{2/3}\sqrt{1 - x^{2/3}}$$

$$y' = \frac{1}{3}x^{-1/3}\,\frac{2 - 3x^{2/3}}{\sqrt{1 - x^{2/3}}}$$

$$y'' = \frac{1}{9}\,\frac{x^{2/3} - 2}{x^{4/3}(1 - x^{2/3})^{3/2}}$$

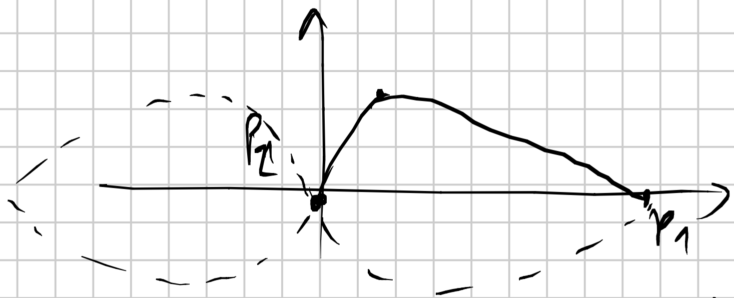
$y' = 0 \implies 2 - 3x^{2/3} = 0 \implies$ $x = \frac{2}{3}\sqrt{\frac{2}{3}}$

$P_3\begin{pmatrix} 0.5443 \\ 0.3849 \end{pmatrix} \implies \varphi = a\cos(\sqrt{r}) \approx 0.8414$ $y = \frac{2}{3}\sqrt{\frac{1}{3}}$

$y'' \leq 0 \implies P_3$ lok max und kurve konkav

$\varphi = 0 \quad P_1(0,0)$

$\varphi = \frac{\pi}{2} \quad P_2\left(0, \frac{\pi}{2}\right)$



$\varphi = t$

$r(t) = \cos^2 t$

$$L = \int_a^b \sqrt{r'(t)^2 + r(t)^2 \varphi'(t)^2}\, dt$$

$r'(t) = -2\cos t \sin t \qquad \varphi' = 1$

$r'(t)^2 + r(t)^2 \varphi'(t)^2 = 4\cos^2 t \sin^2 t + \cos^4 t = \cos^2 t\,(4\sin^2 t + \cos^2 t)$

$$r'(t)^2 + r(t)^2 \varphi'(t)^2 = \cos^2 t\,(1+3\sin^2 t)$$

$$L = 4\int_0^{\pi/2} \sqrt{\cos^2 t\,(1+3\sin^2 t)}\,dt = 4\int_0^{\pi/2} \sqrt{1+3\sin^2 t}\,\underbrace{\cos t\,dt}_{d(\sin t)} =$$

$u := \sin t$

$$= 4\int_0^1 \sqrt{1+3\underbrace{u^2}_{v^2}}\,du = \frac{4}{\sqrt{3}}\int_0^2 \sqrt{1+v^2}\,dv = \frac{4}{\sqrt{3}}\int_0^{\operatorname{arsinh} 2} \cosh^2 z\,dz =$$

$\sqrt{3}u =: v$

$v = \sinh z$

$\sqrt{1+v^2} = \sqrt{\cosh^2 z} = \cosh z$

$dv = \cosh z$

$$= \frac{4}{\sqrt{3}}\,\frac{\sinh z \cosh z + z}{2}\Bigg|_0^{\operatorname{arsinh} 2} =$$

$$= \frac{2}{\sqrt{3}}\operatorname{arsinh} 2$$

Fläche

$$F = \frac{1}{2} \int_{\alpha}^{\beta} r^2(\varphi)\, d\varphi$$

$$F = 4 \cdot \frac{1}{2} \int_0^{\frac{\pi}{2}} (\cos^2 \varphi)^2\, d\varphi = 2 \int_0^{\pi/2} \cos^4 \varphi\, d\varphi = 2 \int_0^{\frac{\pi}{2}} \cos^2 \varphi\,(1 - \sin^2 \varphi)\, d\varphi$$

$$= 2 \left[\underbrace{\int_0^{\frac{\pi}{2}} \cos^2 \varphi\, d\varphi}_{I_1} - \int_0^{\frac{\pi}{2}} \cos^2 \varphi \sin^2 \varphi\, d\varphi \right] = 2 \left[\underbrace{\frac{\sin \varphi \cos \varphi + \varphi}{2} \Big|_0^{\frac{\pi}{2}}}_{I_1} - \frac{1}{4} \int_0^{\frac{\pi}{2}} \sin^2(2\varphi)\, d\varphi \right]$$

$$= 2 \left[\ldots - \frac{1}{8} \int_0^{\pi} \sin^2 t\, dt \right] = 2 \left[\ldots - \frac{1}{8}\, \frac{t - \cos t \sin t}{2} \Big|_0^{\pi} \right]$$

$$= \frac{3\pi}{8}$$

$$X_S = \frac{\iint_F x\, r\, dr\, d\varphi}{F}$$

$$Y_S = \frac{\iint_F y\, r\, dr\, d\varphi}{F}$$

$$\iint_F \cos^3\varphi \cdot \cos^2\varphi \, dr\, d\varphi = \int_0^{2\pi} \int_0^{\cos\varphi} \cos^5\varphi \, d\varphi =$$

$$= \int_0^{2\pi} \cos^7\varphi \, d\varphi = \quad = 0$$