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$$f(x,y) = \begin{cases} \dfrac{\sin^2(xy)}{y} & y \neq 0 \\ 0 & y = 0 \end{cases}$$

(a) Für (a,b) mit $b \neq 0$: „Stetig an Theorie"

Für $(a,0)$: $\forall\ (x_n, y_n) \xrightarrow[n\to\infty]{} (a,0)$ $\quad \lim\limits_{n\to\infty} f(x_n\, y_n) = f(a,0) = 0$

$\Updownarrow$ Stetig in $(a,0)$

$$f(x_n\, y_n) = \begin{cases} \dfrac{\sin^2(x_n y_n)}{y_n} & y_n \neq 0 \\ 0 & \text{sonst} \end{cases}$$

$$\forall x_n \neq 0 \qquad \frac{\sin^2(x_n y_n)}{y_n} = \frac{\sin^2(x_n y_n)}{(x_n y_n)^2}\, x_n^2 y_n$$

$$f(x_n, y_n) = \left(\frac{\overbrace{\sin(x_n y_n)}^{t}}{x_n y_n}\right)^2 \cdot x_n^2 y_n \xrightarrow[x_n \to a,\ y_n \to 0]{} 1^2 \cdot a^2 \cdot 0 = 0 = f(a,0)$$

$$\boxed{\lim_{t \to 0} \frac{\sin t}{t} = 1} \Rightarrow \lim_{\substack{x_n \to a \\ y_n \to 0}} \left(\frac{\sin(x_n y_n)}{x_n y_n}\right)^2 = 1$$

stetig für $(a,0)$ $a \in \mathbb{R}$, d.h. stetig überall in $\mathbb{R}^2$

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$$f_x(x,0) \overset{\text{Def}}{=} \lim_{h\to 0} \frac{\overbrace{f(x+h,0)}^{0} - \overbrace{f(x,0)}^{0}}{h} = \lim_{h\to 0} 0 = 0 \quad \forall x$$

$$f_y(x,0) \overset{\text{Def}}{=} \lim_{h\to 0} \frac{f(x,0+h) - \overbrace{f(x,0)}^{0}}{h} = \lim_{h\to 0} \frac{\frac{\sin^2(x\cdot h)}{h}}{h}$$

$$= \begin{cases} \lim_{h\to 0} 0 = 0 & x = 0 \\ \lim_{h\to 0} \underbrace{\frac{\sin^2(x h)}{h^2\cdot x^2}}_{\to 1} \cdot x^2 = x^2 & x \neq 0 \end{cases}$$

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(an Punkt b)

$$f_{y,x}(0,0) \overset{Def}{=} \lim_{h\to 0} \frac{\overbrace{f_y(0+h,0)}^{=h^2} - \overbrace{f_y(0,0)}^{=0}}{h} = (f_y)_x(0,0)$$

$$= \lim_{h\to 0} \frac{h^2}{h} = \lim_{h\to 0} h = 0$$

b) $$D_v f\left(\frac{\pi}{2},0\right) = \left\langle \operatorname{grad} f\begin{pmatrix} \frac{\pi}{2} \\ 0 \end{pmatrix}, \frac{\vec{v}}{\|\vec{v}\|} \right\rangle =$$

für $x = \frac{\pi}{2}$, $y = 1$

$$v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad f_x\left(\frac{\pi}{2}, 1\right) = \frac{1}{\cancel{y}} \cancel{y}\, 2\sin(xy)\cos(xy) = \sin(2xy) = 0$$

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$$f_y(x,y) = \frac{\left(\overbrace{2\sin(xy)\cos(xy)\cdot x}^{=0}\right) y - \sin^2(xy)}{y^2} \quad \text{für } y \neq 0$$

$$f_y\left(\frac{\pi}{2}, 1\right) = -\frac{\sin^2\left(\frac{\pi}{2}\cdot 1\right)}{1} = -1$$

$$\partial_v f\left(\frac{\pi}{2}, 1\right) = \left\langle \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \cdot \frac{1}{\sqrt{v_1^2 + v_2^2}} \right\rangle = -\frac{v_2}{\sqrt{v_1^2 + v_2^2}}$$