Tut. 13.5.2011, Bsp. 1

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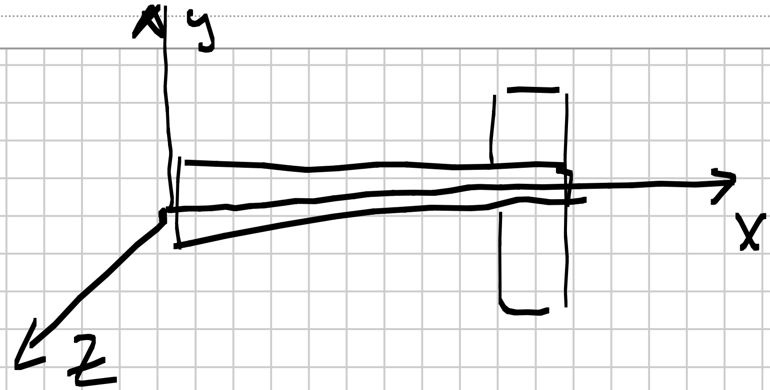

$(S_x, S_y, S_z) = ?$

ρ_H – Holzdichte

ρ_S – Stahldichte

Aus Symmetrie: $S_y = 0$ $S_z = 0$ $S_x = ?$

$$S_x = \frac{\text{Masse}_{\text{kopf}} \cdot S_x^{\text{kopf}} + \text{Masse}_{\text{Stiel}} \cdot S_x^{\text{Stiel}}}{\text{Masse}_{\text{kopf}} + \text{Masse}_{\text{Stiel}}}$$

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$$\text{Masse}_{\text{Stiel}} = \iiint_V \rho_H(x\,y\,z)\,dx\,dy\,dz = \rho_H \iiint_V dx\,dy\,dz = \rho_H\,\text{Vol}_{\text{Stiel}}$$

Fehler in den VO-Unterlagen

$$\rho_x^{\text{Stiel}} = \frac{\rho_H \iiint_V x\,dV}{\text{Masse}_{\text{Stiel}}} = \frac{\cancel{\rho_H} \iiint_V x\,dV}{\cancel{\rho_H}\,\text{Vol}_{\text{Stiel}}} = \frac{\iiint_V x\,dV}{\text{Vol}_{\text{Stiel}}}$$

$$\rho_x^{\text{Kopf}} = \frac{\iiint_{V_{\text{Kopf}}} x\,dV}{\iiint_{V_{\text{Kopf}}} dV} = \frac{\iiint_{V_{\text{Kopf}}} x\,dV}{\text{Vol}_{\text{Kopf}}}$$

$$S_x = \frac{\rho_S \iiint_{V_{Kopf}} x\,dV + \rho_H \iiint_{V_{Stiel}} x\,dV}{\text{Masse}_{Kopf} + \text{Masse}_{Stiel}}$$

Zylinderkoord.

$$\text{Masse}_{Stiel} = \rho_H \int_{\varphi=-\pi}^{\pi} \int_0^{l_1} \int_0^{r_1 - \frac{a}{l_1^2}x^2} r\,dr\,dx\,d\varphi =$$

$$= \rho_H\, 2\pi \int_0^{l_1} \frac{r^2}{2}\Big|_0^{r_1 - \frac{a}{l_1^2}x^2} dx = \pi \rho_H \int_0^{l_1} \left(r_1 - \frac{a}{l_1^2}x\right)^2 dx$$

$$\text{Masse}_{\text{Stiel}} = \rho_H \, \pi \, l_1 \left(r_1 - \frac{2}{3} a r_1 + \frac{a^2}{5} \right) = 91.73 \cdot \rho_H \left(\cancel{cm}^3 \cdot \frac{kg}{\cancel{cm}^3} \right)$$

$$\overline{\text{Masse}_{\text{kopf}}} = ?$$

$$\text{Masse}_k = \text{Masse}_{\text{Quader}} - \text{Masse}_{\text{Bohrung}}$$

$$\text{Masse}_{\text{Quader}} = \rho_S \cdot l_2 \, b_2 \cdot h_2$$

$$\text{Masse}_{\text{Bohrung}} = \rho_S \int_{-\pi}^{\pi} \int_{l_1 - h_2}^{l_1} \int_{0}^{r_1 - \frac{a}{l_1^2} x} r \, dr \, dx \, d\varphi$$

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$$\text{Masse}_{Boh} = \int_S \pi \left(r_1^2 x - 2a \frac{r_1}{l_1^2} \frac{x^3}{3} + \frac{a^2}{5} \frac{x_1^5}{l_1^4} \right) \Bigg|_{l_1 - l_2}^{l_1} \approx$$

$$\approx 79.4516 \text{ cm}^3 \int_S (kg / cm^3)$$

Stiel

$$\iiint_{V_{Stiel}} x \, dV = \int_{-\pi}^{\pi} \int_0^{l_1} \int_0^{r_1 - \frac{a}{l_1^2} x^2} x \; r \, dr \, dx \, d\varphi =$$

$$= \pi \int_0^{l_1} x \frac{r^2}{2} \Bigg|_0^{r_1 - \frac{a}{l_1^2} x^2} dx =$$

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$$= \pi l_1^2 \left(\frac{r_1^2}{2} - \frac{a}{2} r_1 + \frac{a^2}{6} \right) = 1149.8$$

$$\iiint_{V_{Kopf}} x \, dV = \iiint_{V_{Quader}} x \, dV - \iiint_{V_{Bohrung}} x \, dV$$

$$\iiint_{Quader} x \, dV = \int_{-b_2/2}^{b_2/2} \int_{-h_2/2}^{h_2/2} \int_{l_1-h_2}^{l_1} x \, dx \, dy \, dz = b_2 h_2 \left. \frac{x^2}{2} \right|_{l_1-h_2}^{l_1}$$

$$= 398.1312$$

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$$\iiint\limits_{\text{Bohrung}} x\,dV = \int_{-\pi}^{\pi}\int_{l_1-h_2}^{l_1}\int_{0}^{r_1-\frac{a}{l_1^2}x^2} \underline{\underline{x}}\; r\,dr\,dx\,d\varphi =$$

$$= \pi \int_{l_1-h_2}^{l_1} x \left.\frac{r^2}{2}\right|_{0}^{r_1-\frac{a}{l_1^2}x^2} dx =$$

$$= \pi \int_{l_1-h_2}^{l_1} x\left(r_1 - \frac{a}{l_1^2}x^2\right) dx = \pi l_1^2\left(\frac{r_1^2}{2} - \frac{a}{4} r_1 + \frac{e^2}{6}\right) -$$

$$- \pi\left[\frac{r_1^2\,(l_1-h_2)^2}{2} - \frac{a}{2l_1^2}(l_1-h_2)^4 + \frac{a^2}{6l_1^4}(l_1-h_2)^6\right]$$

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$$\iiint_{Bohr} x\,dV = 144.3789$$

$$\iiint_{Kopf} x\,dV = 398.1312 - 144.3789 = \underline{\underline{253.7523}}$$

$$S_x = \frac{\rho_H \; 1149.8 + \rho_S \; 253.75}{49.45 \cdot \rho_S + \rho_H \; 91.73}$$