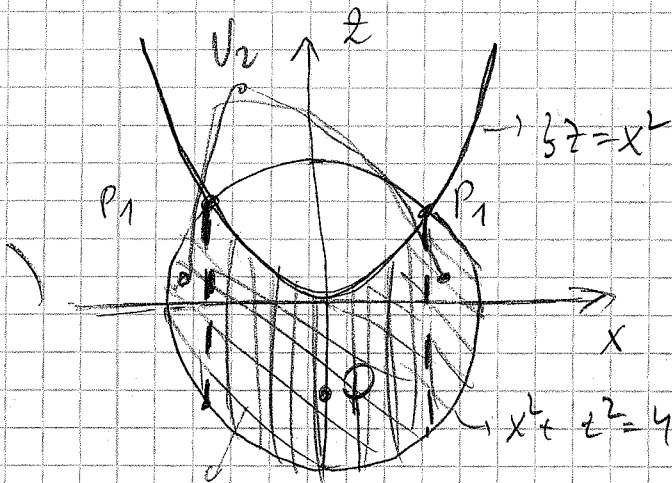


$$2) \quad x^2 + y^2 + z^2 = 4$$

$$x^2 + y^2 = 3z$$

$$P(0, 0, -1)$$

Schnitt mit der  $xOz$  Ebene



$$P_1: \begin{cases} x^2 + z^2 = 4 \\ x^2 = 3z \end{cases} \Rightarrow 3z + z^2 = 4$$

$$z^2 + 3z - 4 = 0$$

$$z = \frac{-3 \pm \sqrt{9 + 16}}{2} \begin{cases} 1 \\ -4 \end{cases}$$

$V_1 \quad z = -4$  Wird ausgeschlossen, weil außerhalb des Kreises  $x^2 + z^2 = 4$

$$z = 1 \Rightarrow x^2 = 3z \Rightarrow 3 \Rightarrow x = \pm \sqrt{3}$$

$$\text{D.h. } P_1 = (\sqrt{3}, 0, 1) \quad P_2 = (-\sqrt{3}, 0, 1)$$

Zylinderkoordinaten  $\delta$

$$x = r \cos \varphi$$

$$y = r \sin \varphi$$

$$z = z$$

$$dv = r dz dr d\varphi$$

Gleichungen der Kugel und des Paraboloids:

$$r^2 + z^2 = 4$$

$$3z = r^2$$

$$\text{Grenzen: } 0 \leq \varphi \leq 2\pi \quad 0 \leq r \leq 2$$

$$\text{Für } 0 \leq r \leq \sqrt{3}: -\sqrt{4-r^2} \leq z \leq \frac{r^2}{3}$$

$$\text{Für } \sqrt{3} \leq r \leq 2: \frac{r^2}{3} \leq z \leq \sqrt{4-r^2}$$

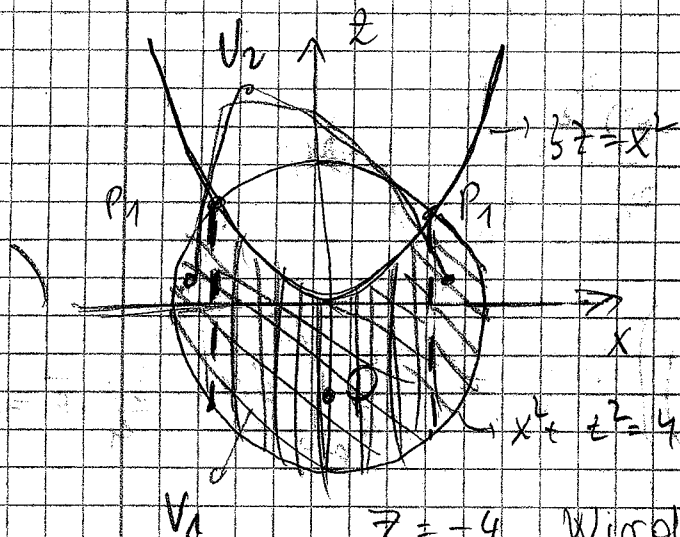
$$V = V_1 + V_2 = \int_{\varphi=0}^{2\pi} \int_{r=0}^{\sqrt{3}} \int_{z=-\sqrt{4-r^2}}^{\frac{r^2}{3}} r dz dr d\varphi + \int_{\varphi=0}^{2\pi} \int_{r=\sqrt{3}}^2 \int_{z=\frac{r^2}{3}}^{\sqrt{4-r^2}} r dz dr d\varphi$$

$$2) \quad x^2 + y^2 + z^2 = 4$$

$$x^2 + y^2 = 3z$$

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Schnitt mit der  $xOz$  Ebene



$$P_1: \quad \left. \begin{array}{l} x^2 + z^2 = 4 \\ x^2 = 3z \end{array} \right\} \Rightarrow 3z + z^2 = 4$$

$$z^2 + 3z - 4 = 0$$

$$z = \frac{-3 \pm \sqrt{9 + 16}}{2} = \frac{-3 \pm 5}{2}$$

$z = -4$  wird ausgeschlossen, weil außerhalb des Kreises  $x^2 + z^2 = 4$

$$z = 1 \Rightarrow x^2 = 3z \Rightarrow 3 \Rightarrow x = \pm \sqrt{3}$$

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Zylinderkoordinaten  $\delta$

$$x = r \cos \varphi$$

$$y = r \sin \varphi$$

$$z = z$$

$$dV = r dr d\varphi dz$$

Gleichungen des Kugel- u. des Paraboloids

$$r^2 + z^2 = 4$$

$$3z = r^2$$

Grenzen:

$$0 \leq \varphi \leq 2\pi$$

$$0 \leq r \leq 2$$

$$\text{Für } 0 \leq r \leq \sqrt{3}: -\sqrt{4-r^2} \leq z \leq \frac{r^2}{3}$$

$$\text{Für } \sqrt{3} \leq r \leq 2: \frac{r^2}{3} \leq z \leq \sqrt{4-r^2}$$

$$V = V_1 + V_2 = \int_{\varphi=0}^{2\pi} \int_{r=0}^{\sqrt{3}} \int_{z=-\sqrt{4-r^2}}^{\frac{r^2}{3}} r dz dr d\varphi + \int_{\varphi=0}^{2\pi} \int_{r=\sqrt{3}}^2 \int_{z=\frac{r^2}{3}}^{\sqrt{4-r^2}} r dz dr d\varphi$$

$$V_1 = \int_{\varphi=0}^{2\pi} \int_{r=0}^{\sqrt{3}} \int_{z=-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r \, dz \, dr \, d\varphi = \int_{\varphi=0}^{2\pi} \int_{r=0}^{\sqrt{3}} r \cdot z \Big|_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} dr \, d\varphi$$

$$= \int_{\varphi=0}^{2\pi} \int_{r=0}^{\sqrt{3}} r \left( \frac{r^2}{3} + \sqrt{4-r^2} \right) dr \, d\varphi = \int_{\varphi=0}^{2\pi} \frac{1}{2} \int_{r=0}^{\sqrt{3}} \left( \frac{r^2}{3} + \sqrt{4-r^2} \right) d(r^2) \, d\varphi$$

$$= \int_{\varphi=0}^{2\pi} \frac{1}{2} \left( \frac{(r^2)^2}{6} \Big|_0^{\sqrt{3}} + \frac{(4-r^2)^{3/2}}{3/2} \Big|_0^{\sqrt{3}} \right) d\varphi =$$

$$= \int_{\varphi=0}^{2\pi} \left( \frac{9}{18} - \frac{1}{2} \left( \frac{2}{3} - \frac{2 \cdot 8}{3} \right) \right) d\varphi = \frac{33}{18} \int_{\varphi=0}^{2\pi} d\varphi = \frac{33}{9} \cdot \pi = 3 \frac{2}{3} \pi$$

$$V_2 = \int_{\varphi=0}^{2\pi} \int_{r=\sqrt{3}}^2 \int_{z=-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r \, dz \, dr \, d\varphi = \int_{\varphi=0}^{2\pi} \int_{r=\sqrt{3}}^2 r \cdot z \Big|_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} dr \, d\varphi$$

$$= \int_{\varphi=0}^{2\pi} \int_{r=\sqrt{3}}^2 2r\sqrt{4-r^2} \, dr \, d\varphi = \int_{\varphi=0}^{2\pi} - \int_{r=\sqrt{3}}^2 \sqrt{4-r^2} d(-r^2) \, d\varphi =$$

$$= \int_{\varphi=0}^{2\pi} - \frac{(4-r^2)^{3/2}}{3/2} \Big|_{\sqrt{3}}^2 \, d\varphi = \int_{\varphi=0}^{2\pi} \frac{2}{3} d\varphi = \frac{2}{3} \cdot 2\pi = \frac{4}{3} \pi = 1 \frac{1}{3} \pi$$

$$V = V_1 + V_2 = 3 \frac{2}{3} \pi + 1 \frac{1}{3} \pi = 5\pi$$

3)  $P_1 = \left(\frac{\pi}{2}, \frac{\pi}{2}, 0\right)$   $P_2 = \left(-\frac{\pi}{2}, \frac{\pi}{2}, 0\right)$   $P = (2\pi, 0, -\pi)$

$$x(t) = \begin{pmatrix} 2t \\ t \sin t \\ t \cos t \end{pmatrix}$$

$$\dot{x}(t) = \begin{pmatrix} 2 \\ \sin t + t \cos t \\ \cos t - t \sin t \end{pmatrix}$$

$$\|\dot{x}(t)\|^2 = 4 + \sin^2 t + t^2 \cos^2 t + 2t \sin t \cos t + \cos^2 t + t^2 \sin^2 t - 2t \cos t \sin t = 5 + t^2$$

$$L = \int_{P_1}^{P_2} ds = \int_{t = -\frac{\pi}{2}}^{\frac{\pi}{2}} \|\dot{x}(t)\| dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{5+t^2} dt =$$

$$x\left(\frac{\pi}{2}\right) = \begin{pmatrix} 2 \cdot \frac{\pi}{2} \\ \frac{\pi}{2} \sin \frac{\pi}{2} \\ \frac{\pi}{2} \cos \frac{\pi}{2} \end{pmatrix} = \begin{pmatrix} \pi \\ \frac{\pi}{2} \\ 0 \end{pmatrix} = P_1$$

$$x\left(-\frac{\pi}{2}\right) = \begin{pmatrix} 2 \cdot \left(-\frac{\pi}{2}\right) \\ -\frac{\pi}{2} \sin\left(-\frac{\pi}{2}\right) \\ -\frac{\pi}{2} \cos\left(-\frac{\pi}{2}\right) \end{pmatrix} = \begin{pmatrix} -\pi \\ \frac{\pi}{2} \\ 0 \end{pmatrix} = P_2$$

Variablen substitution:

$$t = \sinh u \quad dt = \cosh u \, du$$

$$1+t^2 = 1+\sinh^2 u = \cosh^2 u$$

$$\sqrt{1+t^2} = \sqrt{\cosh^2 u} = \cosh u$$

$$L = \int \cosh^2 u \, du = - \left[ \sinh u \cosh u \Big|_0^a - \int_0^a \sinh^2 u \, du \right] =$$

$$\cosh(0) = 1$$

parabell

$$x = \cosh u$$

$$y = \sinh u$$

$$= - \left[ \sinh u \cosh u \Big|_0^a - \int_0^a (\cosh^2 u - 1) \, du \right]$$

$$= \left[ \sinh u \cosh u + a - \int_0^a \cosh^2 u \, du \right]$$

Daraus folgt:

$$2 \int_0^a \cosh^2 u \, du = \sinh a \cosh a + a$$

$$\int_0^a \cosh^2 u \, du = \frac{\sinh a \cosh a + a}{2}$$

Somit gilt  $L = - \frac{\sinh a \cosh a + a}{2}$

$L = -2.0792$  (negativ weil die Kurve negativ orientiert von  $P_1$  mit  $t = \frac{\pi}{2}$  nach  $P_2$  mit  $t = -\frac{\pi}{2}$ )

Länge im physikalischen Sinne = 2.0792

Krümmung am P:  $k = \frac{\sqrt{\|\dot{x}\|^2 \|\ddot{x}\|^2 - \langle \dot{x}, \ddot{x} \rangle^2}}{\|\dot{x}\|^3}$

$P = (2\bar{u}, 0, -\bar{u})$

$$\ddot{x} = \begin{pmatrix} 0 \\ \cos t + \cos t - t \sin t \\ -\sin t - \sin t - t \cos t \end{pmatrix}$$

$$\dot{x}(2\bar{u}) = \begin{pmatrix} 2 \\ 2\bar{u} \\ 1 \end{pmatrix} \quad \ddot{x}(2\bar{u}) = \begin{pmatrix} 0 \\ 2 \\ -2\bar{u} \end{pmatrix}$$

$$\|\dot{x}(2\bar{u})\|^2 = 5 + 4\bar{u}^2 \quad \|\ddot{x}(2\bar{u})\|^2 = 4 + 4\bar{u}^2 = 4(1 + \bar{u}^2)$$

$$\langle \dot{x}, \ddot{x} \rangle = (4\bar{u} - 2\bar{u}) = 2\bar{u}$$

$$k(2\bar{u}) = \frac{\sqrt{(5 + 4\bar{u}^2) 4(1 + \bar{u}^2) - 4\bar{u}^2}}{(5 + 4\bar{u}^2)^{3/2}} = \frac{\sqrt{4\bar{u}^4 + 36\bar{u}^2 + 20 - 4\bar{u}^2}}{(5 + 4\bar{u}^2)^{3/2}}$$

$$k(2\bar{u}) = \frac{2\sqrt{\pi^4 + 8\bar{u}^2 + 5}}{(5 + 4\bar{u}^2)^{3/2}} \approx 0.0908$$

$$\mathcal{G}(\bar{u}) = \frac{(\dot{x}, \ddot{x}, \dddot{x})}{\| \dot{x} \|^2 \| \ddot{x} \|^2 - \langle \dot{x}, \ddot{x} \rangle^2} \Big|_{t=2\bar{u}}$$

$$\ddot{x} = \begin{pmatrix} 0 \\ -2\sin t - \sin t - t \cos t \\ -2\cos t - \cos t + t \sin t \end{pmatrix} \quad \ddot{x}(2\bar{u}) = \begin{pmatrix} 0 \\ -2\sqrt{3} \\ -3 \end{pmatrix}$$

$$\left. \begin{pmatrix} \dot{x}, \ddot{x}, \dddot{x} \\ t=2\bar{u} \end{pmatrix} \right|_{t=2\bar{u}} = \begin{pmatrix} 2 & 0 & 0 \\ 2\bar{u} & 2 & -2\bar{u} \\ 1 & -2\bar{u} & -3 \end{pmatrix} = \begin{pmatrix} 2(-3 - 4\bar{u}^2) \\ -2(4\bar{u}^2 + 3) \end{pmatrix}$$

$$\mathcal{G}(\bar{u}) = \frac{-2(4\bar{u}^2 + 3)}{(5 + 4\bar{u}^2)4(1 + \bar{u}^2) - 4\bar{u}^2} = \frac{2(4\bar{u}^2 + 3)}{\cancel{4}_2(\pi^4 + 8\pi^2 + 5)}$$

$$\mathcal{G}(2\bar{u}) = -0.2342$$