Game Theory WS 2013/2014

8. Exercise Sheet

26. Which of the following games, where Player I is the row player and Player II is the column player, are strategically equivalent to two-player zero-sum games? For each game that is strategically equivalent to a two-player zero-sum game, write explicitly the positive affine transformation which proves your answer.

	L	R
T	11,2	5,4
B	-7,8	17,0

Game A

	L	R
T	2,7	4,5
B	6,3	-3,12

Game B

27. This exercise presents the example of a strategic form game with an infinite set of players that has no equilibrium in mixed strategies. Let $G = (N, (S_i)_{i\in N}, (u_i)_{i\in N})$ be a game in strategic form in which the set of players is the set of natural number $N = \mathbb{N}$, each player $i \in \mathbb{N}$ has two pure strategies $S_i = \{0, 1\}$, and player i's payoff function is given as

$$u_i(s_1, s_2, \ldots, s_n, \ldots) = \begin{cases} s_i & \text{if } \sum_{j\in\mathbb{N}} s_j < \infty \\ -s_i & \text{if } \sum_{j\in\mathbb{N}} s_j = \infty \end{cases} .$$

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(a) Prove that this game has no equilibrium in pure strategies.

(b) Prove that this game has no equilibrium in mixed strategies by using Kolmogorov's $0-1$ law formulated below. (You don't need to prove Kolmogorov's law!)

Let X_i be a sequence of independent random numbers defined over the probability space $(\Omega, \mathcal{F}, p)$. An event A is called a tail event, iff it depends only on $(X_i)_{i\geq n}$ for each $n \in \mathbb{N}$. In other words, for any $n \in \mathbb{N}$, in order to ascertain whether $\omega \in A$, it suffices to know the value of $(X_i(\omega))_{i\geq n}$, which means that we can ignore a finite number of the initial variables X_1, X_2, ..., X_n, for any n. Kolmogorov's $0-1$ law says that *the probability of a tail event is either* 0 *or* 1.