Game Theory WS 2013/2014

6. Exercise Sheet

21. Consider a mixed extension of a strategic form game. A mixed strategy σ_i of player i is called *weakly dominated* if there exists a mixed strategy $\hat{\sigma}_i$ of player i satisfying

    (i) for each vector of pure strategies s_{-i} of the other players, $s_{-i} \in S_{-i}$, the inequality $U_i(\sigma_i, s_{-i}) \leq U_i(\hat{\sigma}_i, s_{-i})$ holds, and

    (ii) there exists a vector of pure t_i of strategies of the other players, $t_{-i} \in S_{-i}$, such that $U_i(\sigma_i, t_{-i}) < U_i(\hat{\sigma}_i, t_{-i})$ holds.

    (a) Show that the set of weakly dominated mixed strategies is a convex set.

    (b) Suppose that player i has a pure strategy s_i which is chosen with positive probability in *each* of his maximin strategies. Prove that s_i is not weakly dominated by any other strategy (pure or mixed).

    (c) Suppose that player i has a pure strategy s_i which is chosen with positive probability in *one* of his maximin strategies. Is s_i chosen with positive probability in each of player i's maximin strategies? Prove this claim or provide a counterexample.

22. Consider the following two-player game where the row player is Player I and the column player is Player II.

	L	C	R
T	6,2	0,6	4,4
M	2,12	4,3	2,5
B	0,6	10,0	2,2

    (a) Verify that no pure strategy is dominated by some other pure strategy in this game.

    (b) Verify that strategy M of Player I is strictly dominated by some mixed strategy.

    (c) Reduce the game by eliminating M and show that in the reduced game strategy R of Player II is strictly dominated by some mixed strategy.

    (d) Reduce the game again by eliminating strategy R. Show that the resulting game has no pure strategy equilibria and determine all its mixed strategy equilibria.

    (e) Determine the mixed strategy equilibria of the original game.

23. A strategic form game $G = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$ is called *symmetric* if (a) each player has the same set of stratgies, $S_i = S_j$, for each $i, j \in N$, and (b) the payoff functions satisfy

    $$u_i(s_1, s_2, \ldots, s_n) = u_j(s_1, \ldots, s_{i-1}, s_j, s_{i+1}, \ldots, s_{j-1}, s_i, s_{j+1}, \ldots, s_n)$$

    for any vector of pure strategies $(s_1, s_2, \ldots, s_n) \in S$ and for each pair of players $i, j \in N$ with $i < j$.

    Prove that in every symmetric game there exists a *symmetric equilibrium* in mixed strategies, i.e. an equilibrium $\sigma = (\sigma_i)_{i \in N}$ satisfying $\sigma_i = \sigma_j$ for each $i, j \in N$.