Game Theory WS 2013/2014

4. Exercise Sheet

15. A two-player zero-sum game on the unit square
Consider the following two-player zero-sum game in strategic form, where:
   - the strategy set of Player I is $X = [0,1] \subset \mathbb{R}$.
   - the strategy set of Player II is $Y = [0,1] \subset \mathbb{R}$.
   - the payoff function (which equals the amount Player II pays to Player I) is given as

$$u(x,y) = 4xy - 2x - y + 3\,, \forall x \in [1,1]\,, \forall y \in [0,1]\,.$$

This game is called *a game on the unit square* because the set of strategy vectors is the unit square in $\mathbb{R}^2$. Check whether this game has a value. If the game does have a value, then identify the optimal strategies for each of the two players. Finally determine all the Nash equilibria of this game.

16. A two-player non-zero-sum game on the unit square
Consider the following two-player non-zero-sum game in strategic form, where:
   - the strategy set of Player I is $X = [0,1] \subset \mathbb{R}$.
   - the strategy set of Player II is $Y = [0,1] \subset \mathbb{R}$.
   - the payoff function of Player I is given as

$$u_I(x,y) = 3xy - 2x - 2y + 2\,, \forall x \in [0,1]\,, \forall y \in [0,1]\,,$$

   - the payoff function of Player II is given as

$$u_{II}(x,y) = -4xy + 2x + y\,, \forall x \in [0,1]\,, \forall y \in [0,1]\,.$$

   (a) Determine the maxmin value and all maxmin strategies for each player in this game.
   (b) Determine all Nash equilibria of this game. By comparing this result to the result obtained in (a) conclude that in two-player non-zero-sum games the concepts of Nash equilibrium and optimal strategies (i.e. maxmin strategies) differ.

17. A function $f\colon [0,1] \times [0,1] \to \mathbb{R}$ is called a *bilinear function*, if the functions $f^{(1)}_{x_0}\colon [0,1] \to \mathbb{R}\,, \mathrm{y} \mapsto \mathrm{f}(\mathrm{x}_0, \mathrm{y})$ and $f^{(2)}_{y_0}\colon [0,1] \to \mathbb{R}\,, \mathrm{x} \mapsto \mathrm{f}(\mathrm{x}, \mathrm{y}_0)$, are affine-linear functions $\forall x_0 \in [0,1]$ and $\forall y_0 \in [0,1]$, respectively. Prove that every two-player zero-sum game with a bilinear payoff function over the unit square is the mixed extension of a two-player game in which every player has two pure strategies. Is the later a two-player **zero-sum** game? Apply this result to the game of Example 15 and determine the corresponding two-player game with two pure strategies for each player.