

Game Theory WS 2013/2014

2. Exercise Sheet

6. Cournot¹ duopoly competition

Two manufacturers labelled 1 and 2 produce the same product and compete for the same market of potential customers. The manufacturers simultaneously select their production quantities, with demand determining the market price of the product, which is identical to both manufacturers. Denote by q_1 and q_2 the quantities of the product produced by manufacturers 1 and 2, respectively. Assume that when the supply is $q_1 + q_2$ the price of each item is $2 - q_1 - q_2$. Assume also that the production cost per item is c_i for manufacturer i , $i \in \{1, 2\}$. Does there exist an equilibrium of this game? If yes, what is it?

7. In a first price auction each buyer submits his bid in a sealed envelope. The winner of the auction is the buyer who submits the highest bid, and the amount he pays is equal to what he bids. If several buyers have submitted bids equal to the highest bid, a fair lottery is conducted among them to choose one winner.

- (a) In this situation, does the strategy v_i^* of buyer i , in which he bids his private value for the item, weakly dominate all his other strategies?
- (b) Does there exist a Nash equilibrium of this game?

8. In the following three-players game, player I chooses a row (A or B), player II chooses a column (a or b) and player III chooses a matrix (α , β or γ). Find all the equilibria of this game.

	a	b
A	0,0,5	0,0,0
B	2,0,0	0,0,0
	α	

	a	b
A	1,2,3	0,0,0
B	0,0,0	1,2,3
	β	

	a	b
A	0,0,0	0,0,0
B	0,5,0	0,0,4
	γ	

9. (a) Find an example of a game $G = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$ in strategic form such that the game \widehat{G} derived from G by elimination of one strategy in one player's strategy set has an equilibrium that is not an equilibrium in the game G . Can you find an analogous example where the eliminated strategy is a dominated strategy of the corresponding player?
- (b) Find an example of a strategic form game G and of an equilibrium s^* of that game such that for each player $i \in N$ the strategy s_i^* is dominated.

¹Antoine Auguste Cournot, 1801-1877, was a french philosopher and mathematician considered to be the father of the modern economic analysis.