Game Theory WS 2013/2014

11. Exercise Sheet

33. Answer the following questions related to David Gale's game of Chomp (see the lecture):

    (a) Which of the two players has a winning strategy in a game of Chomp played on a $2 \times \infty$ board? Justify your answer. Describe the winning strategy.

    (b) Which of the two players has a winning strategy in a game of chomp played on a $m \times \infty$ board, where m is any finite integer? Justify your answer. Describe the winning strategy.

    (c) Find two winning strategies for Player I in a game of Chomp played on an $\infty \times \infty$ board.

34. **The Centipede game**
    The game tree appearing in Figure 1 depicts a two-player game in extensive form (note that the tree is shortened, there are other 94 choice vertices and 94 leaves which are not shown in the figure). The payoffs appear as pairs (x, y) where x is the payoff to Player I (in thousands of dollars) and y is the payoff to Player II (in thousands of dollars). The players make moves in alternating turns with Player I making the first move.

    Every player hat a till into which money is added throughout the play of the game. At the root of the game Player I's till contaions 1000 \$ and Player II's till is empty. Every player in turn, at his move can elect whether to stop the game (S), in which case every player receives as a payoff the amount of money in his till, or to continue to play. Each time a player elects to continue the game, he removes 1000 \$ from his till and places them in the other player's till, while simultaneously the game-master adds another 2000 \$ to the other player's till. If no player has stopped the game after 100 turns have passed, the game ends and each player receives the amount of money in his till at that point. How would you play this game in the role of Player I? Justify your answer?

35. **And-Or**
    And-Or is a two-player game played on a rooted binary tree of depth n, see Figure 3. Every player in turn chooses a leaf of the tree that hat has not previously been selected, and assigns to it the value 1 or 0. After all the leaves have been assigned a value, a value for the entire tree is calculated as in the figure. The first step involves calculating the value of the vertices at one level above the level of the leaves; the value of each such vertex is calculated by using the logic "or" function operating on the values assigned to its children. Next a value is calculated for each vertex one-level up, with that value calculated using the logic "and" function, operating on the values previously calculated for the respective children. The values of all the vertices of the tree are alternately calculated in this manner recursively, with the value of each vertex calculated using either the "and" or the "or" function, operating on values previously calculated for its respective children. Player I wins if the value of the root vertex is 1, and looses if the value of the root vertex is 0. Figure 3 shows the end of a play of this game and the calculation of the vertex values; here Player I wins. Answer the following questions:

    (a) Which player has a winning strategy in a game played on a tree of depth two?

    (b) Which player has a winning strategy in a game played on a tree of depth $2k$, where k is any positive integer?

36. Describe the game Rock-Paper-Scissors as an extensive form game (this game was defined as a strategic form game in the lecture).

37. **Tic-Tac-Toe**
    How many strategies has got Player I in Tic-Tac-Toe, a game in which two players play on a 3×3 board, as depicted in Figure 2? Player one makes the first move and each player in turn chooses a square that has not been selected previously. Player I places an X in every square that he chooses, and Player II places an O in every square that he chooses. The game ends when every square has

been selected. The fisrt player who has managed to place his mark in three adjoining squares, where these three squares form either a row, or a column, or a diagonal, is the winner. (Do not attempt to draw a full game tree. Despite the fact that the rules of the game are quite simple, the game tree is exceedingly large.) Show that each of the two players has a strategy which ensures at least a draw in every play of the game.

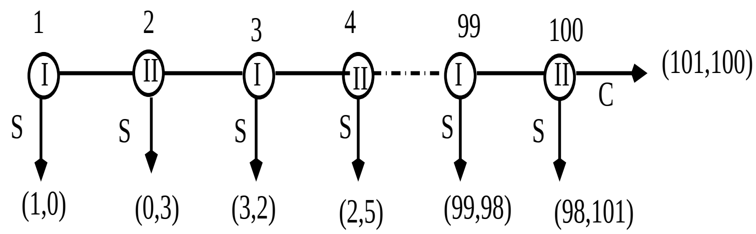

Figure 1: Game-tree for Exercise 34. The unit quantity for the payoffs depicted at the terminal nodes is 1000 $

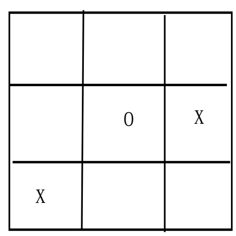

Figure 2: The board of the game Tic-Tac-Toe (Exercise 37) after three moves.

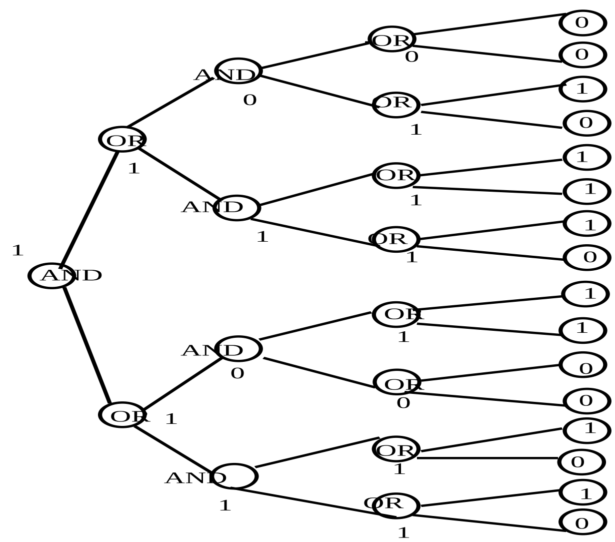

Figure 3: Game-tree for Exercise 35 with depth $n = 4$.