Game Theory WS 2013/2014

10. Exercise Sheet

29. Describe the game of chess as an extensive form game. Is the Theorem of von Neumann (see the lecture) applicable to this game, if we assume that the game of chess is finite, i.e. the number of possible turns is bounded (even if this bound is an astronomically large number)? Is the following statement equivalent to von Neumann's Theorem "The outcome of every play of the game of chess is either a victory for the *white player*, or a victory for the *black player*, or a draw"? Justify your answer.

30. Describe the following situation as an extensive form game. Three piles of matches are on the table. One pile contains a single match, a second pile contains two matches, and the third pile contains three matsches. Two players alternatively remove matches from the table. In each move, the player who is in turn to act at that move may remove matches from one and only one pile and must remove at least one match. The player who removes the last match looses the game.

    By drawing arrows on the game tree identify a way that one of the players can guarantee victory.

31. **Nim** Nim is a two-player game in which piles of matches are placed before the players; the number of piles is finite and each pile contains a finite number of matches. Each player in turn chooses exactly one pile, and removes an arbitrary number $n \geq 1$ of matches from the pile he has selected. The player who removes the last match wins the game.

    (a) Does von Neumann's Theorem imply that one of the players must have a winning strategy? Justify your answer!

    The guided questions presented in the following lead at the end to the construction of a winning strategy in the game of Nim.

    First list in a column the number of matches in each pile expressed in base 2. For example if there are 4 piles containing 2, 12, 13 and 21 matches, respectively, list

    $$\begin{array}{r} 10 \\ 1100 \\ 1101 \\ 10101 \end{array}$$

    Next check whether the number of 1s in each column is odd or even. In the above example counting from the right the number of 1s in the first and the fourth column is even, while in the second, third and fifth column the number of 1s is odd. A state in the game is called a *winning state*, if the number of ones in each column is even. So, the game state above is not a winning state.

    (b) Prove that, starting from any state that is not a winning state, it is possible to get to a winning state in one move, i.e. by removing matches from a single pile. For instance, in the above example one could remove 18 matches from the largest pile and obtain a winning state.

    (c) Prove that at a winning state every legal move leads to a non-winning state.

    (d) Explain why at the end of every play the position of the game will be a winning position.

    (e) Explain how it can be identified which player can guarantee victory for himself, given the number of the piles of matches and the number of matches in each pile, and describe that player's winning strategy.

32. Prove that von Neumann's Thweorem holds in games in extensive form with perfect information in which the game tree has a *countable number of vertices*, but the *depth of every vertex* is bounded, i.e. there exists a natural number K that is greater than the length of every path in the game tree.