Risk theory and risk management in actuarial science
Winter term 2016/17

3d work sheet

16. Consider the logarithmic daily returns of the close prises of the Nasdaq Composite index (`^IXIC`) and apply the method of the Hill estimator to analyse their tails. Perform the following steps with three different time intervals: (I) from November 1, 1995 till November 14, 2016, (II) from November 1, 1995 till December 31, 2005, and (III) from January 1, 2006 till November 14, 2016. Compare the obtained results and comment upon your findings.

    (a) Compare the tails of the empirical distribution of the data set to the tails of the exponential ditribution by means of the QQ-plot.

    (b) Compute the Hill estimator for the empirical data. Argue carefully upon your choice of the threshold parameter k based on the inspection of the Hill plot as in the case of the fire insurance example discussed in the lecture.

    (c) Based on the Hill estimator give an estimator for the $\mathrm{VaR}_{0.95}$ and the $\mathrm{VaR}_{0.99}$ of the data set.

    The data can be downloaded from `finance.yahoo.com`: search for the required index (you can well search for the abbreviation given in paranthesis above), click 'Historical Data', update the 'Time Period' and 'Frequency' appropriately, and finally klick on 'Download data'.

17. (Mean excess function)

    (a) Compute the mean excess function $e(u)$, $u \in \mathbb{R}_+$, for the standard exponential distribution G_0 (i.e. the generalized Pareto distribution with parameter $\lambda = 0$, cf. lecture).

    (b) Compute the mean excess function $e(u)$, $u \in \mathbb{R}_+$, for the generalized Pareto distribution $G_{\lambda,\nu,\beta}$ with $\lambda \in (0,1)$ and $\beta > 0$.

18. Compute the condition value at risk CVaR_q, $q > 1$, of the generalized Pareto distribution $G_{\gamma,\nu,\beta}$ with $\gamma \in (0,1)$ and $\beta > 0$ as a function its value at risk $q_p := \mathrm{VaR}_q$.

19. Use the peaks over threshhold (POT) method to analyse the tails of the data described in Exercise 16.

    (a) Argue carefully upon your choice of the threshold parameter k based on the inspection of the plot of the empirical mean excess function (analogously to the case of the fire insurance example discussed in the lecture).

    (b) Maximize the log-likelihood function to obtain estimators for γ and β by using a solver of your choice. Consider the plot of the different values of the estimator $\hat{\gamma}$ of γ in dependence of the threshold parameter k to back your choice for a suitable interval of values of k (cf. the fire insurance example from the lecture).

    (c) Compute estimators for $\mathrm{VaR}_{0.95}$ and $\mathrm{CVaR}_{0.95}$ for the whole interval of reasonable values of k determined in (b). Visualize the dependence of these estimators on k graphically and revise you choice for the interval of values of k, if appropriate.

    (d) Choose a value of k and visualize in one plot the empirical tail distribution and the tail distribution obtained by the POT method. Comment upon your results.