Beispiel 3.2.10 (b) (Nummerierung wie im Skriptum)

L Zufallsvariable

F_L Verteilungsfunktion $F_L(x) = 1-(1+\gamma x)^{-1/\gamma}$ $x \geq 0$

für $\gamma \in (0,1)$

[Dichte : $F_L'(x) = \frac{1}{\gamma}\gamma(1+\gamma x)^{-\frac{1}{\gamma}-1} = (1+\gamma x)^{-\frac{1+\gamma}{\gamma}}$]

braucht man nicht

$VaR_\alpha(L) = \beta \iff F^{\leftarrow}(\alpha) = \beta \iff \alpha = F(\beta) \iff$

$1-(1+\gamma\beta)^{-1/\gamma} = \alpha \iff \beta = \frac{1}{\gamma}\left[(1-\alpha)^{-\gamma}-1\right]$

$$\boxed{VaR_\alpha(L) = \frac{1}{\gamma}\left((1-\alpha)^{-\gamma}-1\right)}$$

$$CVaR_\alpha(L) = \frac{1}{1-\alpha}\int_\alpha^1 VaR_p(L)\,dp = \frac{1}{1-\alpha}\int_\alpha^1 \frac{1}{\gamma}\left[(1-p)^{-\gamma}-1\right]dp$$

$$= \frac{1}{\gamma(1-\alpha)}\left[\int_\alpha^1 (1-p)^{-\gamma}\,dp - p\Big|_\alpha^1\right] =$$

$$= \frac{1}{\gamma(1-\alpha)}\left[\frac{(1-p)^{-\gamma+1}}{-\gamma+1}\Bigg|_\alpha^1 - (1-\alpha)\right] =$$

$$= \frac{1}{\gamma(1-\alpha)}\left[-\frac{(1-\alpha)^{-\gamma+1}}{1-\gamma} - (1-\alpha)\right] =$$

$$= \boxed{\frac{1}{\gamma}\left[\frac{(1-\alpha)^{-\gamma}}{\gamma-1} - 1\right] = CVaR_\alpha(L)}$$