

LB - Liste von Basen; LPX - Liste von Pareto-optimalen Extrempunkten

LPV - Liste von Pareto-optimalen Strahlen (Charakterisiert durch (x_B, r^j) , wobei x_B und r^j jeweils den Extrempunkt und die Richtung darstellen)
Algorithm 5.2 (Multicriteria Simplex Algorithm)

Initialization: Set $LB := \emptyset$, $LPX := \emptyset$, $LPU := \emptyset$.

Step 1 a) Solve the linear program

$$\begin{aligned} & \min e^t \hat{x} \\ & \text{subject to } Ax + I\hat{x} = b \\ & \quad x, \hat{x} \geq 0. \end{aligned}$$

b) If this LP is unbounded or if the optimal solution is nonzero, STOP, $X = \emptyset$. Otherwise go to Step 2 with a basic feasible solution x^0 of MCLP.

Step 2 a) Solve the linear program (D)

$$\begin{aligned} & \min u^t b + w^t C x^0 \\ & \text{subject to } u^t A + w^t C \geq 0 \\ & \quad w \geq e. \end{aligned}$$

b) If (D) is unbounded, STOP, $X_{\text{Par}} = \emptyset$. Otherwise let (u^*, w^*) be an optimal solution and go to c)

c) Solve the linear program $LP(w^*)$

$$\begin{aligned} & \min w^{*t} C x \\ & \text{subject to } Ax = b \\ & \quad x \geq 0. \end{aligned}$$

Add the optimal basis found for $LP(w^*)$ to LB , the corresponding optimal extreme point to LPX and go to Step 3.

Step 3 a) If $LB = \emptyset$ STOP: All Pareto optimal extreme points and unbounded edges are found. Otherwise choose a basis B in LB , remove it from LB , and go to b)

b) For all variables x_j which are nonbasic at basis B solve the linear program (SP)

$$\begin{aligned} & \max e^t v \\ & \text{subject to } Ry - r^j \delta + Iv = 0 \\ & \quad y, \delta, v \geq 0. \end{aligned}$$

If (SP) has an optimal solution do the following. For all feasible pivots with x_j entering,

- perform the pivot,
- add the adjacent efficient basis to LB , if it is new,
- add the corresponding extreme point to LPX , if it is new,
- add the corresponding unbounded Pareto optimal edges emanating from x_B to LPU , if the pivot column has only negative entries.

Go to Step 3 a)