Operations Research WS 2013/2014

7. Exercise sheet - multicriteria optimization

33. Let $[a,b] \in \mathbb{R}$ be a compact interval and the functions $f_i\colon \mathbb{R} \to \mathbb{R}$ convex, $i = 1, 2, \ldots, Q$. Let

$$x_i^m := \min\{x \in [a,b]\colon f_i(x) = \inf_{x \in [a,b]} f_i(x)\} \text{ and}$$

$$x_i^M := \max\{x \in [a,b]\colon f_i(x) = \inf_{x \in [a,b]} f_i(x)\}$$

Use the result of exercise 32 to show the following identities:

$$X_{Par} = \left[\min_{i=1,2,\ldots,Q} x_i^M, \max_{i=1,2,\ldots,Q} x_i^m\right] \bigcup \left[\max_{i=1,2,\ldots,Q} x_i^m, \min_{i=1,2,\ldots,Q} x_i^M\right]$$

$$X_{w-Par} = \left[\min_{i=1,2,\ldots,Q} x_i^m, \max_{i=1,2,\ldots,Q} x_i^M\right]$$

34. Let $X = \{(x_1, x_2) \in \mathbb{R}^2\colon \mathrm{x}_1 \geq 0, \mathrm{x}_2 \geq 0, \mathrm{x}_1 + \mathrm{x}_2 \leq 100, 2\mathrm{x}_1 + \mathrm{x}_2 \leq 150\}$, $f_1(x_1, x_2) = -6x_1 - 4x_2$ and $f_2(x_1, x_2) = -x_1$. Solve the ϵ-*constrained problem* $P_1(\epsilon)$ for $\epsilon = 0$ (see the lecture notes for the definition of ϵ-constrained problems). Use the method of Benson to check whether the optimal solution x^* of $P_1(0)$ is Pareto-optimal to $(X, f, \mathbb{R}^2)/\mathrm{id}/(\mathbb{R}^2, <)$ or not.

35. Solve the problem in example 34 by means of the compromise programming approach. Use $w = (1/2, 1/2)$ and identify the solution of CP_p^w für $p = 1, 2, \infty$.

36. Let $Y = \{y = (y_1, y_2) \in \mathbb{R}^2\colon \mathrm{y}_1 + \mathrm{y}_2 \geq 1, 0 \leq \mathrm{y}_1 \leq 1\}$. Show that $\hat{y} = (0,1) \in Y_{p-eff}$, and that there is no $w \in W^0$ with $\hat{y} \in A(w, \infty, Y)$, if the ideal point y^0 is involved in CP_p^w. (See the lecture notes for the definition of W^0, $A(w, \infty, Y)$ and CP_p^w.)

37. Let $Y = \{y = (y_1, y_2) \in \mathbb{R}_+^2\colon \mathrm{y}_1^2 + \mathrm{y}_2^2 \geq 1\}$. Show the existence of a parameter p, $1 < p < \infty$, such that the following equality holds:

$$Y_{eff} = \cup_{w \in W^0} A(w, p, Y).$$

Use the ideal point y^0 in the definition of $A(w, p, Y)$ and CP_p^w.

38. Let $Y = \{y = (y_1, y_2) \in \mathbb{R}^2\colon \mathrm{y}_1 + \mathrm{y}_2 \geq 1, 0 \leq \mathrm{y}_1 \leq 1\}$. Show that $\hat{y} = (0,1) \in Y_{p-eff}$, and that there is no $w \in W^0$ with $\hat{y} \in A(w, \infty, Y)$, if the ideal point y^0 is used in CP_p^w. (See the lecture notes for the definition of W^0 and $A(w, \infty, Y)$.)