Operations Research WS 2013/2014

5. Exercise sheet - Stochastic inventory models

26. The stochastic one-period inventory model is extended as follows. There are two products in inventory. They can serve as a substitute for each-other, i.e. Product 1 (Product 2) can be used to fulfill the demand on Product 2 (Product 1), in the case that the demand cannot be fulfilled by the inventory of the specific product. Assume for simplicity that the inventory level of each product equals 0 at the beginning of the planning horizon. Moreover assume that both the fixed ordering costs and the costs of missing demand equal 0. Let c_i be the variable ordering cost per unit of product i and let s_i be the profit realized by fulfilling the demand of one unit of product i, $i = 1, 2$. Hence the fulfillment of a demand for z units of product i results in a net profit of $z(s_i - c_i)$, $i = 1, 2$, for the inventory holder. Let X_i be a random variable representing the demand on product i and let f_i be the probability density function of X_i, $i = 1, 2$. Further let $a_i \in (0, 100)$ be the percentage of customers which would eventually accept a substitute for their demand. Denote by $G(u_1, u_2)$ the (random) profit of the inventory holder if he orders u_i units of product i, $i = 1, 2$. The goal is to determine the amounts u_i of products to be ordered, $i = 1, 2$, such that the expected profit $E(G(u_1, u_2))$ is maximized. Derive a formula for $E(G(u_1, u_2))$.

27. Consider a three-period stochastic stationary inventory model as the one dealt with in the lecture (version A and B). Let the random demand R be uniformly distributed over the interval $[0, 10]$. The other input parameters of the problem are as follows: $c = 1$ Euro per product unit, $K = 5$ Euro, $h = 3$ Euro per product unit and period, $p = 4$ Euro per product unit and period, and $\alpha = 0.9$. Determine an optimal ordering policy $z_j^*(x)$, $x \in \mathbb{R}$, $j = 1, 2, 3$, and the corresponding costs, for each of the models A and B (cf. lecture). Determine also the lower and upper bounds for the ordering point and the ordering level in every period.

28. Consider a stochastic stationary inventory model with an infinite number of periods as described in the lecture. Let the random demand R be uniformly distributed over the interval $[0, 10]$. The other input parameters of the problem are as follows: $c = 1$ Euro per product unit, $K = 0$ Euro, $h = 3$ Euro per product unit and period, $p = 4$ Euro per product unit and period, and $\alpha = 0.9$. Determine an optimal ordering policy $z^*(x)$, $x \in \mathbb{R}$ and the corresponding costs.