Eine Zusammenfassung des selbst-dualen Simplexverfahren (Aus R. J. Vanderbei, Linear Programming Foundations and Extensions, Kluwer, 2001.)

(1) *Identify a spanning tree*—any one will do (see Exercise 13.14). Also identify a root node.

(2) Compute *initial primal flows* on the tree arcs by assuming that nontree arcs have zero flow and the total flow at each node must be balanced. For this calculation, the computed primal flows may be negative. In this case, the initial primal solution is not feasible. The calculation is performed working from leaf nodes inward.

(3) Compute *initial dual values* by working out from the root node along tree arcs using the formula

$$y_j - y_i = c_{ij},$$

which is valid on tree arcs, since the dual slacks vanish on these arcs.

(4) Compute *initial dual slacks* on each nontree arc using the formula

$$z_{ij} = y_i + c_{ij} - y_j.$$

Again, some of the z_{ij}'s might be nonnegative. This is okay (for now), but it is important that they satisfy the above equality.

(5) *Perturb* each primal flow and each dual slack that has a negative initial value by adding a positive scalar μ to each such value.

(6) *Identify a range* $\mu_{\text{MIN}} \leq \mu \leq \mu_{\text{MAX}}$ over which the current solution is optimal (on the first iteration, μ_{MAX} will be infinite).

(7) *Check the stopping rule:* if $\mu_{\text{MIN}} \leq 0$, then set $\mu = 0$ to recover an optimal solution. While not optimal, perform each of the remaining steps and then return to recheck this condition.

(8) *Select an arc* associated with the inequality $\mu_{\text{MIN}} \leq \mu$ (if there are several, pick one arbitrarily). If this arc is a nontree arc, then the current pivot is a *primal pivot.* If, on the other hand, it is a tree arc, then the pivot is a *dual pivot.*
   (a) If the pivot is a primal pivot, the arc identified above is the *entering arc.* Identify the associated *leaving arc* as follows. First, add the entering arc to the tree. With this arc added, there must be a cycle consisting of the entering arc and other tree arcs. The leaving arc is chosen from those arcs on the cycle that go in the opposite direction from the entering arc and having the smallest flow among all such arcs (evaluated at $\mu = \mu_{\text{MIN}}$).
   (b) If the pivot is a dual pivot, the arc identified above is the *leaving arc.* Identify the associated *entering arc* as follows. First, delete the leaving arc from the tree. This deletion splits the tree into two subtrees. The entering arc must bridge these two trees in the opposite direction to the leaving arc, and, among such arcs, it must be the one with the smallest dual slack (evaluated at $\mu = \mu_{\text{MIN}}$).

(9) *Update primal flows* as follows. Add the entering arc to the tree. This addition creates a cycle containing both the entering and leaving arcs. Adjust the flow on the leaving arc to zero, and then adjust the flows on each of the other cycle arcs as necessary to maintain flow balance.

(10) *Update dual variables* as follows. Delete the leaving arc from the old tree. This deletion splits the old tree into two subtrees. Let $\mathcal{T}_u$ denote the subtree containing the tail of the entering arc, and let $\mathcal{T}_v$ denote the subtree containing its head. The dual variables for nodes in $\mathcal{T}_u$ remain unchanged, but the dual variables for nodes in $\mathcal{T}_v$ get incremented by the old dual slack on the entering arc.

(11) *Update dual slacks* as follows. All dual slacks remain unchanged except for those associated with nontree arcs that bridge the two subtrees $\mathcal{T}_u$ and $\mathcal{T}_v$. The dual slacks corresponding to those arcs that bridge in the same direction as the entering arc get decremented by the old dual slack on the entering arc, whereas those that correspond to arcs bridging in the opposite direction get incremented by this amount.