

Gruppe B

1

$$A = \begin{pmatrix} 1 & 0 & 4 \\ 2 & 2 & 1 \\ -2 & -1 & -4 \end{pmatrix}$$

$$\begin{array}{c|ccc} \text{Eigenwerte} & & & \\ \hline & 1-\lambda & 0 & 4 \\ & 2 & 2-\lambda & 1 \\ & -2 & -1 & -4-\lambda \end{array} = \begin{array}{l} (1-\lambda)(0-\lambda)(-4-\lambda)+1 + \\ 4(-2+2(2-\lambda)) = \end{array}$$

$$= (1-\lambda) (\lambda^2 + 2\lambda - 8 + 1) + 4(4 - 2 - 2\lambda)$$

$$= (1-\lambda) (\lambda^2 + 2\lambda - 7) + 8(1-\lambda) =$$

$$= (1-\lambda) (\lambda^2 + 1\lambda + 1) = (1-\lambda) (\lambda + 1)^2 = 0$$

$\lambda_1 = 1$ " $\lambda_2 = -1$ zweifach

Algebraische Vielfachheit von $\lambda_1 = 1$ ist 2
 $\lambda_2 = -1$ ist 2

Eigenvektoren zu $\lambda_1 = 1$

$$\begin{pmatrix} 1-1 & 0 & 4 \\ 2 & 2-1 & 1 \\ -2 & -1 & -4-1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Matrix Inversionen:
 Vertausche 1,2 mit 2,2
 $z, z \leftarrow 3,2 + 1, z$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$z, z \leftarrow 3,2 + 2, z$

Geometrische Vielfachheit = 1

Eigenvektoren: $\left\{ \begin{pmatrix} -2t \\ t \end{pmatrix} : t \in \mathbb{R} \right\} = \left\{ t \begin{pmatrix} -2 \\ 1 \end{pmatrix} : t \in \mathbb{R} \right\}$

$$2x + 4z = 0 \Rightarrow x = -2z = -2t$$

$$3y - 3z = 0 \Rightarrow y = z = t$$

$$t = t \in \mathbb{R}$$

$$\begin{pmatrix} 2 & 0 & 4 \\ 0 & 3 & -3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$3z \leftarrow 3z + \frac{3}{2}z$$

$$\begin{pmatrix} 2 & 0 & 4 \\ 0 & 3 & -3 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$2z \leftarrow 2z - 1z \quad 3z \leftarrow 3z + 4z$$

$$\begin{pmatrix} 1+1 & 0 & 4 \\ 2 & 2+1 & 1 \\ -2 & -1 & -4+1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Eigenvektoren zu $\lambda_2 = -1$

Geometrische Vielfachheit = 1

Eigenvektoren $\left\{ \begin{pmatrix} -t \\ t \\ 0 \end{pmatrix} : t \in \mathbb{R} \right\} = \left\{ t \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} : t \in \mathbb{R} \right\}$

$$2x + y + z = 0 \Rightarrow x = -\frac{y}{2} = -\frac{z}{2}$$

$$4z = 0 \Rightarrow z = 0$$

$$y = t \in \mathbb{R}$$

$$\text{Abs. Fehler} = T(3.15, 1.1) - f(3.15, 1.1) = 0.0007$$

$$\text{Rel. Fehler} = \left| \frac{T(3.15, 1.1) - f(3.15, 1.1)}{f(3.15, 1.1)} \right| = 0.2\%$$

$$T(3.15, 1.1) = -0.3486$$

$$T(3.15, 1.1) = -1 + 2\pi \cdot 0.1 + \frac{2}{(0.01)^2} + (\pi+2) \cdot 0.1 \cdot 0.01 + \frac{(\pi^2 - 2\pi)(0.1)^2}{2}$$

$$\overline{29} \quad f(3.15, 1.1) = -0.3477$$

$$T(x, y) = -1 + 2\pi(y-1) + \frac{2}{x^2} \left[(x-\pi)^2 + 2(\pi+2)(y-1)(x-\pi) + (\pi^2 - 2\pi)(y-1)^2 \right]$$

$$T(x_0, y_0) = f(x_0, y_0) + f_x(x_0, y_0)(x-x_0) + f_y(x_0, y_0)(y-y_0) + \frac{1}{2} \left[f_{xx}(x_0, y_0)(x-x_0)^2 + 2f_{xy}(x_0, y_0)(x-x_0)(y-y_0) + f_{yy}(x_0, y_0)(y-y_0)^2 \right]$$

$$f_{yy} = -x^2 \cos(xy) - \frac{2x}{y^3}$$

$$f_{yy}(\pi, 1) = \pi^2 - 2\pi$$

$$= -\sin(xy) - xy \cos(xy) + \frac{2}{y}$$

$$f_{xy} = -\sin(xy) - y \cos(xy) + \frac{1}{y^2} \cdot 2xy$$

$$f_{xy}(\pi, 1) = \pi + 2$$

$$f_{xx} = -y^2 \cos(xy)$$

$$f_{xx}(\pi, 1) = 1$$

$$f_y = -x \sin(xy) + 2xy \cdot \frac{1}{y^3} = -x \sin(xy) + \frac{2x}{y^2}$$

$$f_y(\pi, 1) = 2\pi$$

$$f_x = -y \sin(xy) + \ln(y^2)$$

$$f_x(\pi, 1) = 0$$

$$(x_0, y_0) = (\pi, 1)$$

$$f(x, y) = \cos(xy) + x \ln(y^2)$$

$$f(\pi, 1) = -1$$

20)

(3)

$$f(x,y) = \frac{x^2}{64} - \frac{1}{2}x + \frac{1}{4}y$$

lokale Extrema

$$f_x = \frac{x}{32} - \frac{1}{2} = 0$$

$$f_y = \frac{y}{64} - \frac{1}{4} = 0$$

$$x + 0 \neq 0$$

$$x^2 y = -64$$

$$\left(\frac{64}{y^2}\right)^2 y = -64$$

$$y^3 = -64 \Rightarrow y = -4$$

$$x = \frac{16}{64} = 4$$

letzte Resultat $P(4, -4)$

$$f_{xx} = -\frac{1}{32}$$

$$f_{xy} = \frac{1}{64} = f_{yx}$$

$$f_{yy} = \frac{1}{32}$$

$$H_f(4, -4) = \begin{pmatrix} -1/32 & 1/64 \\ 1/64 & 1/32 \end{pmatrix}$$

$$\det H_f(4, -4) = \frac{1}{64} - \frac{1}{64} = 0$$

lok. Maximum

$$f(4, -4) = -\frac{3}{4}$$

$$1 \leq x \leq 9, -5 \leq y \leq -1$$

Globale Extrema

$$X = 1 - 9 \leq y \leq -1$$

$$f(1, 9) = \frac{1}{64} - \frac{1}{2} + \frac{9}{4}$$

$$f(1, 9) = \frac{1}{64} - \frac{1}{2} + \frac{9}{4}$$

$y = 9$ am Rand

Kandidaten Extrema (geben)

$P_1(4, -4)$	$f(4, -4) = -3/4$	$P_1(4, -4)$	$f(4, -4) = -2.0156$
$P_2(4, -8)$	$f(4, -8) = -5/4$	$P_2(4, -9)$	$f(4, -9) = -1.2517$
$P_3(9, -8/3)$	$f(9, -8/3) = -0.8644$	$P_3(9, -1)$	$f(9, -1) = -1.9878$
$P_4(8, -1)$	$f(8, -1) = -1.25$	$P_4(9, -8)$	$f(9, -8) = -1.2517$
$P_5(8/3, -9)$	$f(8/3, -9) = -0.6644$		

glob. Max $P_1(4, -4)$ mit Wert -0.75 ; glob Min $P_6(4, -1)$ mit Wert -2.0156

$$f(x, -8) = -\frac{64}{8} + \frac{1}{4}x^2 = 0 \Rightarrow x = \pm 8$$

$$f''(x, -8) = \frac{2}{4}x = \frac{x}{2} < 0$$

$x = 8$
 \Rightarrow lok. Max
 am Rande

$$f(x, -1) = -\frac{64}{4} + \frac{1}{4}x^2 = 0 \Rightarrow x = \pm 8$$

$$f''(x, -1) = \frac{2}{4}x = \frac{x}{2} < 0$$

$x = 8$
 \Rightarrow lok. Max
 am Rande

$$f(9, y) = \frac{64}{9} + \frac{1}{4} + \frac{1}{4}y^2 = 0 \Rightarrow y = \pm \frac{8}{3}$$

$$f''(9, y) = \frac{2}{4}y = \frac{y}{2} < 0$$

$y = 8/3$
 am lok. Max am
 Rande

$$f(4, y) = \frac{64}{4} + \frac{1}{4} + \frac{1}{4}y^2 = 0 \Rightarrow y = \pm 8$$

$$f''(4, y) = \frac{2}{4}y = \frac{y}{2} < 0$$

$y = -8$
 kritischer Punkt
 \Rightarrow lok. Max. am Rande