(1) Gruppe A

$$A = \begin{pmatrix} 3 & 1 & 4 \\ 4 & 3 & 4 \\ -4 & -2 & -5 \end{pmatrix}$$

Eigenwerte

$$\begin{pmatrix} 3-\lambda & 1 & 4 \\ 4 & 3-\lambda & 4 \\ -4 & -2 & -5-\lambda \end{pmatrix} = \begin{vmatrix} 3-\lambda-4 & 1-3+\lambda & 0 \\ 4 & 3-\lambda & 4 \\ -4 & -2 & -5-\lambda \end{vmatrix}$$

$$= (-1-\lambda)\begin{vmatrix} 3-\lambda & 4 \\ -2 & -5-\lambda \end{vmatrix} - (\lambda-2)\begin{vmatrix} 4 & 4 \\ -4 & -5-\lambda \end{vmatrix}$$

$$= -(\lambda+1)(\lambda^2+2\lambda-15+8) - (\lambda-2)(-20-4\lambda+16)$$

$$= -(\lambda+1)(\lambda^2+2\lambda-7) + 4(\lambda-2)(\lambda+1)$$

$$= (\lambda+1)(4\lambda-8-\lambda^2-2\lambda+7)$$

$$= -(\lambda+1)(\lambda^2-2\lambda+1) = -(\lambda+1)(\lambda-1)^2$$

$\lambda_1 = -1 \qquad \lambda_2 = 1$ zweifach

Algebraische Vielfachheit $\quad \lambda_1 \rightarrow \underline{1}$
$\qquad\qquad\qquad\qquad\qquad\quad\; \lambda_2 \rightarrow \underline{2}$

Eigenvektoren
zu $\lambda_1 = -1$

$$\begin{pmatrix} 3+1 & 1 & 4 \\ 4 & 4 & 4 \\ -4 & -2 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Transformation der Matrix:

2. Zeile − 1. Zeile
3. Zeile + 1. Zeile

$$\begin{pmatrix} 4 & 1 & 4 \\ 0 & 3 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

3. Zeile + $\frac{1}{3}$ 2. Zeile

$$\begin{pmatrix} 4 & 1 & 4 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$z = t \in \mathbb{R}$ $\quad 3y = 0 \Rightarrow y = 0$

$$4x + y + 4z = 0$$
$$\Downarrow$$
$$4x + 0 + 4t = 0$$
$$\Downarrow$$
$$x = -t$$

Eigenvektoren zu λ_1 : $\left\{ \begin{pmatrix} -t \\ 0 \\ t \end{pmatrix} : t \in \mathbb{R} \right\} = \left\{ t \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} , t \in \mathbb{R} \right\}$

Geometrische Vielfachheit von $\lambda_1 = 1$

zu $\lambda_2 = 1$

$$\begin{pmatrix} 3-1 & 1 & 4 \\ 4 & 3-1 & 4 \\ -4 & -2 & -5-1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

2. Zeile − 2x 1. Zeile
3. Zeile + 1. Zeile

$$\begin{pmatrix} 2 & 1 & 4 \\ 0 & 0 & -4 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

3. Zeile − $\frac{1}{2}$ x 2. Zeile

$$\begin{pmatrix} 2 & 1 & 4 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$y = t \in \mathbb{R}$ beliebig

$-4z = 0 \Rightarrow z = 0$

$2x + y + 4z = 0 \Rightarrow x = -\frac{y}{2} = -\frac{t}{2}$

· <u>Eigenvektoren</u> 0,25 $\left\{ \begin{pmatrix} -t/2 \\ t \\ 0 \end{pmatrix} : t \in \mathbb{R} \right\} = \left\{ t \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} : t \in \mathbb{R} \right\}$

Geometrische Vielfachheit $= 1$.

2a) $f(xy) = \sin(xy) + x\ln(y)$

$(x_0, y_0) = (\pi, 1)$

$f_x = y\cos(xy) + \ln y$ $\qquad f_x(\pi, 1) = -1$

$f_y = x\cos(xy) + \frac{x}{y}$ $\qquad f_y(\pi, 1) = -\pi + \pi = 0$

$f_{xx} = -y^2 \sin(xy)$ $\qquad f_{xx}(\pi, 1) = 0$

$f_{xy} = \cos(xy) - xy\sin(xy) + \frac{1}{y}$ $\qquad f_{xy}(\pi, 1) = 0$

$f_{yy} = -x^2 \sin(xy) - \frac{x}{y^2}$ $\qquad f_{yy}(\pi, 1) = \pi$

$f(x_0 y_0) = 0$

$T(xy) - f(x_0 y_0) + f_x(x_0 y_0)(x - x_0) + f_y(x_0 y_0)(y - y_0)$

$+ \frac{1}{2}\left[f_{xx}(x - x_0)^2 + 2 f_{xy}(x_0 y_0)(x - x_0)(y - y_0) + \right.$

$\left. f_y(x_0 y_0)(y - y_0)^2 \right]$

$T(xy) = -(x - \pi) + \frac{1}{2}(-\pi)(y-1)^2 = (\pi - x) - \frac{\pi}{2}(y-1)^2$

2b) $f(3.15, 1.1) = \sin(3.15 \cdot 1.1) + 3.15 \ln(1.1) = -0.0176$

$T(3.15, 1.1) = -0.01 - \frac{\pi}{2}(0.1)^2 = -0.02411$

Absoluter Fehler $\quad T(3.15, 1.1) - f(3.15, 1.1) = -0.00654327$

Relativer Fehler $\quad \left| \frac{T(3.5, 1.1) - f(3.15, 1.1)}{f(3.15, 1.1)} \right| = 37.23\%$

3) $f(x,y) = \frac{xy}{27} + \frac{1}{x} - \frac{1}{y}$ $\quad x \neq 0 \quad y \neq 0$

- lokale Extrema.

$$\left.\begin{aligned} f_x &= \frac{y}{27} - \frac{1}{x^2} = 0 \\ f_y &= \frac{x}{27} + \frac{1}{y^2} = 0 \end{aligned}\right\} \Rightarrow \begin{aligned} yx^2 &= 27 \\ x \cdot y^2 &= -27 \end{aligned} \quad \Rightarrow y = \frac{27}{x^2} \quad x\cdot\left(\frac{27}{x^2}\right)^2 = -27$$

$$\frac{(27)^2}{x^3} = -27 \Rightarrow x^3 = -27 \Rightarrow \boxed{x = -3}$$

$$y = \frac{27}{(-3)^2} = 3$$

kritische Punkte : $P(-3,3)$
~~$P = (-3,3)$~~

$$f_{xx} = \frac{2}{x^3} \qquad f_{yy} = -\frac{2}{y^3} \qquad H_f(xy) = \begin{pmatrix} 2/x^3 & 1/27 \\ 1/27 & -2/y^3 \end{pmatrix}$$

$$f_{xy} = \frac{1}{27} = f_{yx}$$

$$\det H_f(-3,3) = \frac{-4}{(-3)^3 \cdot 3^3} - \frac{1}{27^2} = \frac{3}{27^2} > 0$$

$$\frac{2}{x^3} = \frac{2}{(-3)^3} < 0$$

$\Rightarrow P(-3,3)$ ist ein Maximum.

$f(-3,3) = -\frac{1}{3} + \frac{1}{3} - \frac{1}{3}$

$f(-3,3) = -1$

- $-4 \leq x \leq -2$
  $2 \leq y \leq 4.$

Untersuchung des Randes

$\boxed{2 \leq y \leq 4}$ $\boxed{x = -4}$ $\quad f(-4,y) = \frac{-4y}{27} + \frac{1}{4} - \frac{1}{y}$

$f'(-4,y) = -\frac{4}{27} + \frac{1}{y^2} = 0 \Rightarrow \quad y = \pm\frac{3\sqrt{3}}{2}$

Der negative Wert wird ausgeschlossen weil $y \in [2,4]$

$f''(-4,y) = -\frac{2}{y^3} < 0$ in $[2,4]$

D.h. $y = \frac{2}{3\sqrt{3}}$ ist ein lokales Maximum am Rand $2 \le y \le 4$, $x = -4$.

$\boxed{x = -2}$ $\boxed{2 \le y \le 4}$

$f(-2,y) = -\frac{2y}{27} - \frac{1}{2} - \frac{1}{y}$

$f'(-2,y) = \frac{1}{y^2} - \frac{2}{27} = 0 \Rightarrow$ $y = 3\sqrt{\frac{3}{2}}$

ausgeschl.? $\Rightarrow \frac{2}{3}\sqrt{\frac{2}{3}}$ ist ein Maxim.

$f''(-2,y) = -\frac{2}{y^3} < 0$ in $[2,4]$

$\boxed{y = 2 \quad -4 \le x \le -2}$

$f(x,2) = \frac{2x}{27} + \frac{1}{x} - \frac{1}{2}$

$f'(x,2) = \frac{2}{27} - \frac{1}{x^2} = 0 \Rightarrow x = \pm\frac{3\sqrt{3}}{\sqrt{2}} = 3\sqrt{\frac{3}{2}}$

ausgeschlossen weil $x < 0$

$f''(x,2) = \frac{2}{x^3} < 0 \Rightarrow$ Maximum

$\boxed{y = 4 \quad -4 \le x \le -2}$

$f(x,4) = \frac{4x}{27} + \frac{1}{x} - \frac{1}{4}$

$f'_x(x,4) = \frac{4}{27} - \frac{1}{x^2} = 0 \Rightarrow x = \pm\frac{3\sqrt{3}}{2}$

$f''(x,4) = \frac{2}{x^3} < 0 \Rightarrow$ Maximum ausgeschl.

Kandidaten für Globale Extreme

$P_1(-3,3)$ $f(-3,3) = -1$

$P_2(-4, \frac{3\sqrt{3}}{2})$ $f(-4, \frac{3\sqrt{3}}{2}) = -1.0198$

$P_3(-2, 3\sqrt{\frac{3}{2}})$ $f(-2, 3\sqrt{\frac{3}{2}}) = -1.0443$

$P_4(-3\sqrt{\frac{3}{2}}, 2)$ $f(-3\sqrt{\frac{3}{2}}, 2) = -1.0443$

0.25

$P_5(-3\sqrt{\frac{3}{2}}, 4)$ $f(-3\sqrt{\frac{3}{2}}, 4) = -1.0198$

$P_6(-4,2)$ $f(-4,2) = -1.0463$

$P_7(-4,4)$ $f(-4,4) = -1.0926$

$P_8(-2,2)$ $f(-2,2) = -1.1481$

$P_9(-2,4)$ $f(-2,4) = -1.0463$

Glob. Maximum ist $P_1(-3,3)$ mit Wert -1; Glob. Minimum $P_8(-2,2)$ mit Wert -1.1481