Advanced and algorithmic graph theory
Summer term 2016

4th work sheet

27. Show that a planar graph G is 2-connected iff the border of every face of any geometric dual G^* of G is a cycle.

28. Let G be a planar connected graph. Show that $(G^*)^*$ is isomporphic to G, where G^* is a geometric dual of G and $(G^*)^*$ is a geometric dual of G^*. Illustrate the statement by examples where $\kappa(G) = 1$, $\kappa(G) = 2$ and $\kappa(G) = 3$ hold, respectively.

29. Let G and H be connected graphs.

    (a) Show that G contains H as a minor, i.e. $G \succ H$, iff there exists a mapping $\phi\colon V(G) \to V(H)$ such that the preimages $\phi^{-1}(\{v\})$, for $v \in V(H)$, fulfill the following properties:

    (i) $G[\phi^{-1}(\{v\})]$ is connected, $\forall v \in V(H)$, and
    (ii) $\forall \{u, v\} \in E(H)$ the cut $\delta\Big(\phi^{-1}(\{u\}), \phi^{-1}(\{v\})\Big)$ in G is nonempty.

    (b) If $\Delta(H) \leq 3$, then $G \succ H$ holds iff G contains H as a subdivision.

30. (a) Specify an infinite set of self-dual graphs.

    (b) Show that if a geometric dual G^* of a graph G is 2-connected, than G itself is not necessarily 2-connected. (Recall that G^* is in general a multigraph, i.e. it may contain loops and mutiple edges. The definition of 2-connectivity for multigraphs is analogous to the definition of 2-connectivity for graphs: a multigraph G is 2-connected if and only if the graph obtained from G by removing some arbitrarily chosen vertex $v \in V(G)$ together with the edges incident to v is still connected.)